HOW DOES COURT STABILITY AFFECT LEGAL STABILITY?

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Abstract

The formation of coalitions in a court has attracted attention by political scientists and legal economists. The question we address in this article is the extent to which coalition stability impacts the law. We consider a model where a court has two judicial coalitions, majority and minority. However, they may change their relative influence in time. We show that, while both sides have a preferred legal policy and want their standard to become law, the two coalitions may compromise on not changing the standard because of majority uncertainty in the future. One important implication from our article is that less certainty concerning the future (in terms of majority and minority) could actually make the law more stable in the present (since the standard is unchanged).

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1. INTRODUCTION

There is by now a vast theoretical and empirical literature on judicial behavior, from the attitudinal model to multiple variations on the strategic model (Bergara et al, 2003; Epstein and Knight, 1998; Epstein et al, 2013; Maltzman et al, 2000; Segal and Spaeth, 2002; Spiller and Tiller, 1996). The theoretical models explain how judges make decisions in collegial courts. Starting with a rational theory of what judges maximize (Posner 1993, 2004 & 2010), the different approaches in the literature emphasize varying relevant aspects in order to explain judicial decision-making. They differ in the possible determinants of judicial behavior, from ideology and other political variables to gender, age, religion, professional and social background. Interactions with further actors have been also considered: politicians, lobbyists, regulators, media, lawyers and law audiences, and so on (Segal, 1997; Segal and Westerland, 2005). The empirical literature has broadly provided validity about the theoretical predictions, but it has not been immune to controversies (Epstein and Jacobi, 2010; Cameron and Kornhauser, 2017; Segal et al, 2011).

There is considerably less work at the aggregate level, that is, explaining court behavior as a complex institution rather than the sum of individual judges (some exceptions include Baker and Mezzetti, 2012; Bustos and Jacobi, 2019; Callander and Clark, 2017; Landes and Posner, 1975; Ramseyer and Rasmusen, 1997). Most literature focuses on explaining the role of courts in promoting and protecting judicial independence as well as the interaction of the judiciary with the other two branches of government, legislative and executive (Landes and Posner, 1975). At this level, the empirical literature documents the impact of judicial independence and court performance on economic growth and other macroeconomic variables (Hanssen, 2004; Helmke and Rosenbluth, 2009; La Porta et al, 2008).

The formation of coalitions in a court has attracted attention by political scientists and legal economists given the historical dynamics of the US Supreme Court. The main goals of the literature have been to explain why judges join or configure certain coalitions (Cameron and Kornhauser, 2010; Epstein et al, 2007; Kornhauser, 1992a & 1992b; Segal et al, 1995), the role of the median judge (Clark, 2008; Martin et al, 2005; Segal and Cover, 1989) or coalition survival (Caminker, 1999; Geli and Spiller, 1990 & 1992; Segal 1997). However, how the possibility of changes in court composition affect or shape behavior and legal policy has not been debated extensively. Particularly, at the meso level (that is, above the micro level of individual judges, but below the macro level of courts), there is limited work aimed at clarifying how coalitions interact and affect legal policy (Kornhauser, 1995; Wahlbeck, 1997).

Looking around the world, we know some courts have a more stable composition than others, depending on appointment rules and party system (Garoupa and Ginsburg, 2015). For example, due to the political cycles in the United States, since the early 1950s, the composition between Republicans and Democrats is to some extent balanced and somehow predictable. However, in Brazil, where the US model of judicial appointment has been imported a long time ago (Oliveira and Garoupa, 2011), in part due to mandatory retirement at 70, the composition of the court reflects significant political swings (because a President serving two full terms has the ability to appoint several justices, almost the entire bench). In Argentina, there have been periods of great change (for example, after the democratic transition) and periods of long stability (in the last decade) reflecting cycles in partisanship (González Bertomeu et al, 2016; Helmke, 2005).

In European constitutional courts, we also find some interesting features. Germany has practiced a quota system for many decades (the same institutional mechanism exists in countries like Austria and Portugal) – the main parties have a fixed number of seats and appointments do not reflect short-term parliamentary balance of power (Garoupa and Ginsburg, 2015). Other countries, such as France, Italy or Spain, have less stable composition since it displays changes in the legislative and executive branches of government (Garoupa and Ginsburg, 2015).

The question we address in this article is the extent to which coalition stability impacts the law. We consider a model where a court has two judicial coalitions, majority and minority. However, they may change their relative influence in time. Let us call these coalitions Blue and Red. Blue could be majority in the present and minority in the future; the opposite applies to Red. We show that, while both sides have a preferred legal policy and want their standard to become law, the two coalitions may compromise on not changing the standard because of majority uncertainty. If one coalition polarizes and imposes its own standard, the other side will change the standard in the opposite direction if given the opportunity (hence, inducing a costly "standards-race"). However, if the majority keeps the original standard, the other side is also likely to keep the original standard if given a chance.[1] All of these effects depend on different parameters such as costs amending the law, stability of coalitions, predictability and so on. In particular, results significantly depend on the position of the original standard vis-à-vis the standards favored by both coalitions. In a reminiscence of the logic revealed by the literature that studies U.S. Supreme Court

[1] In other words, the initial majority coalition of a given court will see its expected extra payoff associated to a new standard substantially affected by the probability of remaining as majority in the future.

nominations and confirmations (Moranski and Shipan, 1999; Bustos and Jacobi, 2014), predictions vary conditional on whether coalitions are ideologically aligned or opposed.

One important implication from our article is that uncertainty concerning the future (in terms of majority and minority) could actually make the law more stable (since the standard is unchanged). We uncover a pattern somehow similar to a Rawlsian veil of ignorance narrative – when both coalitions are afraid of possible repercussions in the future from swinging positions (behind the veil of ignorance), they are disciplined in the present and compromise by not making significant changes in the law, thus avoiding a costly "standards-race". The argument is also related to the political insurance theory in the context of judicial independence (Vanberg, 2015) – the possibility of being a minority in the future convinces the current majority to protect judicial independence (in our model, keeping the standard untouched) at the cost of not implementing their most favorite policy. Also there are some commonalities between our results and the seminal article by Stephenson (2003). The latter explains judicial independence review as the outcome of a competitive political system, moderate judicial doctrine and a degree of risk aversion when looking forward. Although our model is about courts, and not the political balance within the executive and legislative branches, the resulting suggestions are aligned. It is political competition (reflected in the composition of the court) and uncertainty about the future that induces moderate judicial doctrines.

The result suggested in our article also matters for the debate on the evolution of the common law (Gennaioli and Shleifer, 2007; Niblett et al, 2010; Parameswaran, 2018; Ponzetto and Fernandez, 2008). In the context of that literature, we show that the role of particular judicial preferences (or possible judicial biases) is affected by uncertainty concerning court composition. In fact, the present varying preferences of the court could actually deter change, therefore, diluting the so-called evolutionary adaptability of common law (as opposed to the result suggested by Gennaioli and Shleifer, 2007) when there is significant uncertainty concerning the prevailing future preferences of the court.

Another area related to our article is the political economy of horizontal stare decisis (Kornhauser, 1989; O'Hara, 1992; Rasmusen, 1994). We derive that horizontal stares decisis happens because both coalitions are too uncertain about the future and prefer to defer to the current standard rather than risking an unwelcome change of standard in the future.

One particular recent article is quite close to ours (Cameron et al, 2019). They focus on horizontal stare decisis and model a heterogeneous bench. In their model, judges have different preferences. These differences create opportunities for gains from trade. Such gains are maximal when all judges agree on applying a common standard. Partial stare decisis emerges when certain commitment restrictions preclude full trade.

Apart from the applications (we are more interested in the stability of the law as a function of the political stability of court, and less in explaining partial stare decisis as a commitment device) and context (we model legal stability as the outcome of a noncooperative game rather than gains in trade from cooperation), there are, however, a few additional important differences. We use a reduced form game with two periods rather than a repeated infinite horizon game (in a sense, our explicit costs of changing the standard reflect implicit costs of trade in Cameron et al, 2019). We rely on uncertainty about future influence (political power changes exogenously) to explain choices rather than using the multiplicity of possible cases to trade on possible consensus. Still, we emphasize a similar insight – against common wisdom, judicial polarization might provide stability rather than undermine it.

Section 2 introduces the model. The solutions to the model are presented in section 3. In section 4 we generalize our results. We discuss the results in section 5. Final remarks are addressed in section 6.

2. THE MODEL

A two-period lived court exercising constitutional review (simply "Court") has the option to change the legal standard at the beginning of each period. We denote $s_0 \in [0,1]$ as the standard initially faced by the Court and $s_t \in \{s_0, b, r\}$ with $t \in \{1,2\}$ as the standard set by the Court in periods 1 and 2 respectively. While legal standards are typically not a continuum, in Section 4, we discuss our results when $s_t \in [0,1]$.

Political coalitions and the standard setting process

The Court is only conformed by justices that belong to either a Red or a Blue coalition. While Red has ideology $r \in [0,1]$, Blue has ideology $b \in [0,1]$. Each period one of the political coalitions is majority and the other coalition is minority. Without loss of generality we impose that $r > b$ where 1 is most conservative and 0 is most liberal. While we know that Blue is majority in the first period, it remains as majority in the second period only with probability x exogenously given.[2]

The majority coalition can freely change the legal standard, the minority coalition cannot stop that process. That is, in the first period Blue sets s_1. Instead, in the second period

[2] This probability captures expectations on changes in the composition of the Court conditional on the political and electoral system.

Blue sets s_2 with probability x but Red sets s_2 with probability $1 - x$. The cost of changing the standard is $c > 0$ regardless whether the majority is Blue or Red.[3]

Pay-offs

The only two players of the game are the Red and Blue coalitions. Without loss of generality, we call $U_i(s) = 1 - (i - s)^2$ the benefit obtained by coalition $i \in \{r, b\}$ when the Court sets standard s such that $U_i(s)$ is concave with $\frac{\partial U_i(i)}{\partial s} = 0$ and $U_r(r) = U_b(b) = 1$. The payoff of each coalition is maximal when the standard imposed by the Court is the coalition's standard. In addition to the benefit that each coalition gets every period because of the state of the law, the majority coalition that changes the standard at the beginning of the period faces cost c for doing that — note that the majority coalition might decide not to change the standard. As usual we denote the discount factor as δ. For parsimony, we introduce the following notation

$$\Delta \equiv U_b(b) - U_b(r) = U_r(r) - U_r(b) = (r - b)^2 \text{ (measures polarization across coalitions)}$$

$$\Delta_i(s) \equiv U_i(i) - U_i(s) = (i - s)^2 \text{ (measures disposition for each coalition)}$$

We define the scenarios: "opposed ideologies" and "aligned ideologies" as

$$\text{opposed ideologies} \equiv b < s_0 < r$$

$$\text{aligned ideologies} \equiv b < r < s_0 \text{ or } s_0 < b < r$$

[3] The cost of changing the standard is the same regardless which coalition is majority because we interpret *c* to be the political cost of changing the law. Main results do not change if we assume that the cost in the first period is a constant but a random variable distributed according to a certain distribution in the second period. The mathematical analysis becomes more cumbersome if the cost is different in both periods, but it does not affect the qualitative results we derive with the basic model.

We also introduce the following definitions for consideration in later sections of the article:

(i) *Maximum Stability*: No coalition changes the initial standard;

(ii) *High Stability*: Only one coalition changes the standard when it takes a particular value;

(iii) *High Instability*: Both coalitions change the standard when it takes a particular value;

(iv) *Maximum Instability*: Both coalitions change any standard that is not their optimum.

3. MAIN RESULTS

3.1. WHEN DOES MAJORITY CHANGE THE STANDARD?

Here we determine the conditions under which Blue prefers to keep the initial standard s_0 or changes it to b in the first period.[4] To do that we first determine the incentives of the coalitions at the second period and then focus in Blue's decision during the first period. As main result, we find that the impact of x over Blue's decision to change the standard is centrally determined by two factors. First, the cost of performing that change and second the ideological position of the initial standard. Here we look at opposed ideologies, in section 4 we discuss the case in which coalitions' ideologies are aligned.

We find that when c is neither too small nor too large, which are extreme scenarios in which Blue always and never changes the standard respectively, it changes the legal standard with a probability that strictly increases with x. In other words, the first period majority coalition costly adjusts the legal standard if the probability that it will remain as majority in the second period is large enough. As we will show later, the reason of this result is that the majority coalition obtains a larger present value pay-off if it is majority in both

[4] The alternative r is dominated by s_0 or b. Recall that we have imposed $s_t \in \{s_0, b, r\}$.

periods and not only in the first period because as majority in the second period it *keeps* the standard it sets in the first period.

Having in mind that $s_1 \in \{s_0, b\}$ we characterize Blue's and Red's decisions in the scenarios in which they are majority at $t = 2$.[5] If Blue is majority then it does not change the standard when $s_1 = b$ but it changes it to $s_2 = b$ when $s_1 = s_0$ conditional on $U_b(b) - c > U_b(s_0)$ or equivalently $c < \Delta_b(s_0)$. Analogously, Red changes the standard at the second period if and only if $c < \Delta_r(s_0)$ when $s_1 = s_0$ and if and only if $c < \Delta_r(b)$ when $s_1 = b$.

Now we can identify Blue's strategy in the first period. In order to do that we need to compare the coalition expected pay-off when it keeps the standard as s_0 or changes it to b. Blue's pay-off when it sets b is

$$U_B(b) = \begin{cases} 1 - c + \delta\big(x + (1-x)(1-\Delta)\big) & \text{if } c < \Delta \\ 1 + \delta - c & \text{if } c \geq \Delta \end{cases}$$

If Blue sets b at t = 1 then a second period Blue majority doesn´t change the standard but a Red majority changes it to r if $c < \Delta$. Note that the pay-off has a discontinuity when $c = \Delta$ because that is the cost that convinces Red not to change standard b.

It should be noted that the more polarized are the coalitions, the larger is the payoff obtained by Blue for changing the standard (larger the set in which c < Δ).

On the other hand, Blue's pay-off when it sets s_0 depends on this same value. Here we impose that the initial standard is close to Blue's ideal point, that is $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$. In the Appendix we discuss all the other cases when $s_0 \in [b, r]$. Recall that in Section 4 we analyze scenarios in which $s_0 < b$ and $s_0 > r$.

Blue's pay-off when it sets s_0 in the first period is given by

[5] Note that in a $T - Periods$ model $s_{T-1} \in \{s_T, b, r\}$ because any of the coalitions can be majority at T-1.

$$U_B(s_0)=\begin{cases}1-\Delta_b(s_0)+\delta((1-c)x+(1-x)(1-\Delta)) \text{ if } c<\Delta_b(s_0)\\ 1-\Delta_b(s_0)+\delta\big(x(1-\Delta_b(s_0))+(1-x)(1-\Delta)\big) \text{ if } c\in[\Delta_b(s_0),\Delta_r(s_0)]\\ (1-\Delta_b(s_0))(1+\delta) \quad \text{if } c>\Delta_r(s_0)\end{cases}$$

The meaning of the previous pay-off is direct. If Blue sets s_0 in the first period then a second period Blue majority changes the standard to b if $c<\Delta_b(s_0)$ and a Red majority changes it to r if $c<\Delta_r(s_0)$. Otherwise the standard remains as s_0. There is a discontinuity when $c=\Delta_r(s_0)$ because the change in Red's decision generates a jump in Blue's pay-off.

We can intuitively observe that with more polarized coalitions, it is less attractive to keep s_0 because both Δ and $\Delta_r(s_0)$ go up.

We separate our discussion on Blue's decision in scenarios in which c takes small, large or intermediate values. If $c<\Delta_b(s_0)$ then Blue always change the standard to b. Indeed the coalition changes the standard if the cost of doing that is smaller than the addition of two effects. First, the immediate benefit from adjusting the standard. Second, the benefit of not having to pay the cost for a standard adjustment in the second period if Blue remains as majority. The effect in the second period if Red becomes a majority is not relevant because regardless whether s_1 is b or s_0 Red always changes it to r as the cost is small enough

$$U_B(b)>U_B(s_0)\leftrightarrow c<\underbrace{\Delta_b(s_0)}_{\text{First Period Effect}}+\underbrace{\delta xc}_{\text{Second Period Effect when Blue is Majority}}$$

Because this inequality can be re-written as

$$c<\frac{\Delta_b(s_0)}{1-\delta x} \qquad (1)$$

it follows that Blue changes the standard for all values of x when $c<\Delta_b(s_0)$. On the other extreme, when $c\geq\Delta$ Blue never changes the standard. To see that, note that Blue sets b if the benefit of having its optimal standard in both periods ($s_1=s_2=b$) and not s_0 is greater than the cost of changing the standard in the first period, that is

$$U_B(b) > U_B(s_0) \leftrightarrow 1 + \delta - c \ \ > (1 - \Delta_b(s_0))(1 + \delta)$$

$$\leftrightarrow c \ \ < (1 + \delta)\Delta_b(s_0) \qquad (2)$$

But (2) never holds because $\Delta > (1 + \delta)\Delta_b(s_0)$.[6] Indeed, given that $\Delta_r(s_0) > (1 + \delta)\Delta_b(s_0)$, (2) implies that for all x, Blue never changes the standard when $c > \Delta_r(s_0)$.[7]

We are only missing the characterization of the most interesting case which takes place when $c \in [\Delta_b(s_0), \Delta_r(s_0)]$. Then Blue sets b when

$$U_B(b) > U_B(s_0) \leftrightarrow 1 - c + \delta\big(x + (1 - x)(1 - \Delta)\big)$$

$$> 1 - \Delta_b(s_0) + \delta\big(x(1 - \Delta_b(s_0)) + (1 - x)(1 - \Delta)\big)$$

$$\leftrightarrow c < \Delta_b(s_0) + \delta x \Delta_b(s_0) = (1 + \delta x)\Delta_b(s_0) \qquad (3)$$

Once more, the coalition changes the standard at the first period only if the cost is smaller than two effects. The cost of a change must be smaller than the addition of the immediate benefit from adjusting the standard and the second period benefit of facing standard b instead of s_0 if Blue is majority — this time a Blue second period majority does not change s_0. The decision of a Red second period majority is still irrelevant because it always sets r.

As a main novelty, (3) tells us that Blue changes the standard only if the probability of staying as the majority coalition at t = 2 is greater than a certain threshold because Blue´s benefit from setting b instead of s_0 increases with the probability to remain as majority. That is, *the higher the probability that Blue keeps the control of the Court, the more likely is that it changes the standard at t = 1*. The threshold can be identified after we re-write (3) as

$$x > \frac{c - \Delta_b(s_0)}{\delta \Delta_b(s_0)} = x^*(c) \to \frac{\partial x^*(c)}{\partial c} = \frac{1}{\delta \Delta_b(s_0)} > 0$$

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[6] It is true that $(r - b)^2 > (1 + \delta)(s_0 - b)^2$ because that inequality is equivalent to $s_0 < \frac{1}{\sqrt{1+\delta}} r + (1 - \frac{1}{\sqrt{1+\delta}}) b$ whis is true because $\frac{1}{1+\sqrt{1+\delta}} < \frac{1}{\sqrt{1+\delta}}$.

[7] Because $s_0 \in \left[b, \frac{r}{1+\sqrt{1+\delta}} + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$ then $\Delta_r(s_0) = (r - s_0)^2 > (1 + \delta)(s_0 - b)^2$.

Intuitively, the more expensive is to change the standard, the greater is the required probability of Blue being majority for that coalition to decide to change the standard Alternatively, the greater the probability of Blue being majority, the larger the set of values of c for which it decides to change the standard. Figure 1.1.O provides a graphical representation of the previous characterization.[8]

<<Insert Figure 1.1.O about here>>

Figures 1.2.O-1.4.O show what is different when s_0 gets closer to r.[9] On one side there are fewer cases (smaller set of costs) in which Blue keeps s_0 (boundary $\overline{c}$ moves to the right). The intuition is that the closer is s_0 to r the least attractive is for Blue to keep that standard for all values of x. On the other side, there are more cases (larger set of costs) in which Blue sets b for all values of x (boundary $\underline{c}$ also moves to the right).[10]

<<Insert Figures 1.2.O-1.4.O about here>>

Note that in Scenario 4 there exists a range of values of c in which $x^*(c) = 0$, it is the reason why $x^*(c)$ is not strictly increasing for all $c \in [\underline{c}, \overline{c}]$. The meaning of this exception is that in Scenario 4, s_0 is so close to r that when $c \in [\Delta, (1+\delta)\Delta_b(s_0)]$ a Red second period majority neither changes standard s_0 nor standard b. Hence Blue prefers to set b than keep s_0 because for all values of x, that coalition obtains $1 - c + \delta$ with the first decision but only

[8] Note that when $c \in [\Delta_b(s_0), (1+\delta)\Delta_b(s_0)]$ then $x^*(c) \in [0,1]$ but when $c \in [(1+\delta)\Delta_b(s_0), \Delta_r(s_0)]$ then $x^*(c) > 1$ which implies that s_0 dominates b.

[9] The move from figure 1.1 to figure 1.2 is due to the change in the inequality: $(r - s_0)^2 < (1+\delta)(s_0 - b)^2 \leftrightarrow s_0 > \frac{r}{1+\sqrt{1+\delta}} + \frac{\sqrt{1+\delta}b}{1+\sqrt{1+\delta}}$. The move from figure 1.2 to figure 1.3 is due to the change in the inequality: $(r - s_0)^2 < (s_0 - b)^2 \leftrightarrow s_0 > \frac{r+b}{2}$. The move from figure 1.3 to figure 1.4 is due to the change in the inequality: $(r-b)^2 < (1+\delta)(s_0 - b)^2 \leftrightarrow s_0 > \frac{r}{\sqrt{1+\delta}} + (1 - \frac{1}{\sqrt{1+\delta}})b$.

[10] The jumps in $x^*(c)$ when $c \in [\underline{c}, \overline{c}]$ are explained by the discontinuities in Blue´s pay-offs when Red changes its strategy in the second period. For example, in figure 1.2.O Blue's pay-off goes up when $c = \Delta_r(s_0)$ because at that cost Red stops changing s_0 to r which makes more attractive for Blue to keep s_0.

obtains $(1+\delta)(1-\Delta_b(s_0))$ with the second decision. In all the other scenarios this case does not exist because $\Delta > (1+\delta)\Delta_b(s_0)$.

Proposition 1 summarizes all the previous cases in a simple statement that emphasizes that *the probability of a change in the law goes up with the probability that there is no-change in the coalition that controls the Court.*

PROPOSITION 1 (Blue's decisions at t = 1 with opposed coalitions):

There exists $\underline{c}, \overline{c}$ *and* $x^*(c)$ *when* $c \in [\underline{c}, \overline{c}]$ *with* $\frac{\partial x^*(c)}{\partial c} \geq 0$ *such that Blue sets*

$$s_1 = \begin{cases} b \text{ when } c < \underline{c} \\ \begin{cases} b \text{ if } x \geq x^*(c) \\ s_0 \text{ if } x < x^*(c) \end{cases} \text{when } c \in [\underline{c}, \overline{c}] \\ s_0 \text{ when } c > \overline{c} \end{cases}$$

Proof: See the Appendix.

3.2. LEGAL STABILITY

In the previous subsection, we focused exclusively in the strategy followed by the first period majority coalition. However, it is also of interest to know under which conditions the legal standard will not change or be, frequently or unfrequently, modified in the future. In order to answer these questions in Corollary 1 we identify the scenarios in which the standard is stable or unstable over time. Later in Corollary 2 we emphasize the conditions under which the standard reaches maximum stability (we have certainty that the standard is never changed).

COROLLARY 1 (Stability of the legal standard with opposed coalitions):

There exist c_1, c_2, c_3, c_4 *such that*

a. *If* $c < c_1$ *then Blue sets* $s_1 = b$. *Later a Blue second period majority keeps* $s_2 = s_1$ *but a Red second period majority sets* $s_2 = r$.

b. *If* $c \in [c_1, c_2]$ *then Blue sets* $s_1 = b$ *if* x *is large enough but keeps* $s_1 = s_0$ *otherwise. Then a Blue second period majority keeps* $s_2 = s_1$ *but a Red second period majority sets* $s_2 = r$.

c. *If* $c \in [c_2, c_3]$ *then Blue sets* $s_1 = b$ *if* x *is large enough but keeps* $s_1 = s_0$ *otherwise. Then a Blue second period majority keeps* $s_2 = s_1$ *but a Red second period majority sets* $s_2 = r$ *conditional on* $s_1 = b$ *otherwise it keeps* $s_2 = s_1 = s_0$.

d. *If* $c \in [c_3, c_4]$ *then Blue sets* $s_1 = b$. *Later a Blue or a Red second period majority keeps* $s_2 = s_1 = b$.

e. *If* $c > c_4$ *then Blue keeps* $s_1 = s_0$. *Later a Blue or a Red second period majority keeps* $s_2 = s_1 = s_0$.

Proof: See the Appendix.

As we show in the Appendix, depending on whether s_0 is closer to b or to r, some of the scenarios disappear.[11] Several conclusions follow from Corollary 1.

If the cost to change the standard is very small then the majority coalition, regardless whether it is during the first or the second period, sets its optimal standard if that is not already the case. That is, the standard shows *maximum instability*. Analogously, if the cost to change the standard is very large then the majority coalition, regardless whether it is the first or the second period, never changes the standard. That is, the standard shows *maximum stability*.

[11] When s_0 is closer to b then $c_3 = c_4$ and when s_0 is closer to r then $c_2 = c_3$.

If the cost to change the standard has a small intermediate value ($c \in [c_1, c_2]$) and the initial standard is closer to Blue's preferences than Red's preferences, then Blue changes the standard if and only if x is large enough. A Blue second period majority preserves the standard and a Red second period majority always changes the first period standard. That is, the standard shows *high instability*.

If the cost to change the standard has a large intermediate value ($c \in [c_2, c_3]$) such that Blue changes the standard if and only if x is large enough then a Red second period majority only changes the standard if Blue imposed its preferred ideology in the first period. That is, the larger the probability that Blue remains as majority then the closer we are to a scenario in which the standard presents *maximum instability*. Even more, if the probability that Blue remains as majority is small enough then the standard shows *maximum stability* because Blue does not want to set b as that triggers that Red sets it back to r and Red does not want to change s_0 to r because the benefit is not large enough to compensate the cost of doing so.

Finally, if the cost to change the standard has a large intermediate value ($c \in [c_3, c_4]$) and the initial standard is closer to Red's preferences than Blue's preferences then Blue changes the standard in the first period and the second period majority, regardless whether it is Red or Blue, keeps the standard set by Blue.[12] That is, the standard shows *high stability*.

Summing up, the stability of a legal standard depends on the interaction of costs and the likelihood of changes in court majority. There are two extreme cases that do not offer any particular surprise – very high costs induce *maximum stability* and very low costs induce *maximum instability*. Therefore, we should focus on intermediate and probably more realistic

[12] As we show in the Appendix, it has to be that $\frac{b+r}{2} < \frac{1}{\sqrt{1+\delta}} b + (1 - \frac{1}{\sqrt{1+\delta}}) r < s_0 < r$.

costs. *Maximum stability* is still a possibility as long as the probability of Blue keeping a majority is not too high. Why? The reason is that Blue is afraid of changing the standard and losing a majority in the following period, opening the way to Red imposing their own standard (which is worse than keeping the original standard). Corollary 2 uncovers the details of all scenarios in which opposed coalitions never change the standard.

COROLLARY 2 (Maximum stability of the legal standard with opposed coalitions):

There exist $\underline{s_0}, \overline{s_0}$ *also* $\overline{c}$, $\overline{\overline{c}}$, $\overline{\overline{\overline{c}}}$ *and* $x^*(c)$ *with* $\frac{\partial x^*(c)}{\partial c} > 0$ *such that no coalition ever changes the standard*

a. *If* $s_0 < \underline{s_0}$: *For all* $c > \overline{c}$.

b. *If* $s_0 \in [\underline{s_0}, \overline{s_0}]$: *For all* $c \in [\overline{c}, \overline{\overline{c}}]$ *when* $x \leq x^*(c)$ *and for all* $c > \overline{\overline{c}}$.

c. *If* $s_0 > \overline{s_0}$: *For all* $c \in [\overline{c}, \overline{\overline{c}}]$ *when* $x \leq x^*(c)$ *and for all* $c > \overline{\overline{\overline{c}}}$.

Proof: See the Appendix.

Corollary 2 tells us that *maximum stability* not only takes place when the cost of a change is very large but it also takes place when the cost takes intermediate values. However the last is possible only when the probability that Blue remains as majority is small enough, $x \leq x^*(c)$. A low x not only leads Blue to keep the initial standard during the first period (to avoid the costly standard race) but the not so small cost convinces a Red majority in the second period not to change the standard as well . As we show in the next section (subsection 4.2), a second period Red majority that expects a final period (3-period game) does change standard s_0 if x is very small. In a symmetric way, Red changes the standard in the second period only if it does not expect a Blue third period majority to change it back. The result is that in games of more than two periods, *maximum stability* takes place when c and x both take intermediate values!

4. EXTENSIONS

4.1. COALITIONS WITH ALIGNED IDEOLOGIES

Consider now the case of aligned ideologies. Although it is still true that Blue always (never) change the standard when the cost is small (large enough), results are somehow distinct when the cost has an intermediate value. *If the initial standard is to the right of both Blue and Red ideal points then the probability of Blue remaining as majority in the second period becomes irrelevant to the strategy followed by Blue in the first period*. The reason is that the payoff that Blue gets in the second period when it sets b always dominates the second period payoff when it keeps s_0. Hence the only scenario in which Blue keeps the standard is when the cost of a change is greater than the benefit of not facing the suboptimal standard s_0 in both periods which is $(1+\delta)\Delta_b(s_0)$ and does not depend on x. Instead if the initial standard is to the left of both Blue and Red ideal points then the first period majority coalition follows a strategy in which it adjusts the legal standard if x is large enough *or follows a strategy in which it adjusts the legal standard if x is small enough*. The second strategy is an option when ideologies are aligned but not when are opposed because under alignment Blue's future expected benefit because of imposing its preferred ideology can be larger if Red and not Blue is majority in the second period. That is not possible when the coalitions have opposed ideologies, Blue is always better off when it remains as the majority if it chooses to change the standard in the first period.

In order to better understand the result in which the initial standard is very conservative ($s_0 > r$), suppose that $\Delta_b(s_0) > \Delta_r(s_0) > \Delta$ (the case in which $\Delta_b(s_0) > \Delta > \Delta_r(s_0)$ is analogous) such that $U_B(b)$ is as we wrote down in 3.1. but $U_B(s_0)$ is given by

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c < \Delta_r(s_0) \\ 1 - \Delta_b(s_0) + \delta\big(x(1-c) + (1-x)(1-\Delta_b(s_0))\big) \text{ if } c \in [\Delta_r(s_0), \Delta_b(s_0)] \\ (1 - \Delta_b(s_0))(1+\delta) \quad \text{if } c > \Delta_b(s_0) \end{cases}$$

It is direct to verify that Blue sets the standard equal to b for any value of x when $c < \Delta$ and keeps s_0 for any value of x when $c > \Delta_b(s_0)$.[13] The questions is, what happens if $c \in [\Delta, \Delta_b(s_0)]$? The answer unravels after Blue compares

$$U_B(b) = 1 - c + \delta$$

with

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c \in [\Delta, \Delta_r(s_0)] \\ 1 - \Delta_b(s_0) + \delta\big(x(1-c) + (1-x)(1-\Delta_b(s_0))\big) \text{ if } c \in [\Delta_r(s_0), \Delta_b(s_0)] \end{cases}$$

Blue prefers to set b given that the cost is smaller than $\Delta_b(s_0)$ and in the second period the standard remains as b regardless what coalition is majority in the second period, which does not happen if Blue keeps s_0 in the first period. The difference with the opposed ideologies scenario is that now, Δ is always smaller than $\Delta_b(s_0)$ which means that the initial standard is so far away that Blue has strong incentives to change it in the first period.[14] In addition, when the cost takes intermediate values ($c \in [\Delta, \Delta_b(s_0)]$), Red does not have incentives to change the standard b because the cost of a change is, relatively speaking, too high.

On the other side, in order to better understand the result in which the initial standard is very liberal ($s_0 < b$), suppose that $\Delta_r(s_0) > \Delta > \Delta_b(s_0)$ such that $U_B(s_0)$ is given by

[13] If $c < \Delta$ then evidently $1 - c + \delta\big(x + (1-x)(1-\Delta)\big) > 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta))$ and if $c > \Delta_b(s_0)$ then $(1 - \Delta_b(s_0))(1+\delta) > (1 - \Delta_b(s_0))(1+\delta) \leftrightarrow c > (1+\delta)\Delta_b(s_0)$.

[14] Blue even prefers standard r to s_0.

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c < \Delta_b(s_0) \\ 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big) \text{ if } c \in [\Delta_b(s_0), \Delta_r(s_0)] \\ (1-\Delta_b(s_0))(1+\delta) \quad \text{if } c > \Delta_r(s_0) \end{cases}$$

Once more Blue sets b for all values of x when c is small enough ($c < \Delta_b(s_0)$) and keeps s_0 for all values of x when c is large enough ($c > \Delta_r(s_0)$). However, this time new forces are at play when $c \in [\Delta_b(s_0), \Delta_r(s_0)]$. If $c \in [\Delta_b(s_0), \Delta]$ then like in the opposed ideologies scenario, Blue sets b only when x is large enough. Indeed, Blue sets b if:

$$1 - c + \delta\big(x + (1-x)(1-\Delta)\big) > 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big)$$

$$\leftrightarrow c < (1+\delta x)\Delta_b(s_0) \leftrightarrow x > \frac{c - \Delta_b(s_0)}{\delta \Delta_b(s_0)}$$

which is exactly (3). That is, the probability that Blue sets its ideal point increases with x because that allows that majority to capture larger benefits as a second period majority when compared to the scenario in which it keeps s_0. But if $c \in [\Delta, \Delta_r(s_0)]$ then Blue sets b only if x is small enough! Now Blue sets b if and only if:

$$1 + \delta - c > 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big) \qquad (4)$$

$$\leftrightarrow c < (1+\delta x)\Delta_b(s_0) + \delta(1-x)\Delta \leftrightarrow x < \frac{(\Delta_b(s_0)+\delta\Delta) - c}{\delta(\Delta - \Delta_b(s_0))}$$

For first time in all the cases that we have analyzed, Blue sets b only when the probability to remain as majority in the second period is small enough. Blue increases its payoff associated to b relative to s_0 when the likelihood that Red becomes the new majority also increases. As revealed by (4), the second period difference in payoff when Blue sets b (which both coalitions keep in the second period) versus s_0 (which Blue keeps but Red changes to r in the second period) when Blue is majority is $x\delta\Delta_b(s_0)$ but the same difference when Red is majority is $(1-x)\delta\Delta$. Because $\Delta > \Delta_b(s_0)$ it follows that Blue wins more by setting b if at t = 2 Red is majority instead of Blue. That is not possible when ideologies are opposed.

<<Insert Figures 1.1.A-1.3.A about here>>

While figures 1.1.A-1.3.A present Blue decisions at t = 1 when ideologies are aligned graphically, Proposition 2 present them mathematically. In the Appendix we provide a detailed proofs.

PROPOSITION 2 (Blue's decisions at t = 1 with aligned coalitions):

i. *When* $s_0 < 2b - r < b < r$ *then there exists* $\underline{c}$, $\bar{c}$ *and* $x^*(c)$ *when* $c \in [\underline{c}, \bar{c}]$ *with* $\frac{\partial x^*(c)}{\partial c} > 0$ *such that Blue sets*

$$s_1 = \begin{cases} b \text{ when } c < \underline{c} \\ \begin{cases} b \text{ if } x \geq x^*(c) \\ s_0 \text{ if } x < x^*(c) \end{cases} \text{when } c \in [\underline{c}, \bar{c}] \\ s_0 \text{ when } c > \bar{c} \end{cases}$$

ii. *When* $2b - r < s_0 < b < r$ *then there exists* $\underline{c}$, $\bar{c}, \bar{\bar{c}}$ *and* $x^*(c_1)$ *when* $c \in [\underline{c}, \bar{c}]$ *with* $\frac{\partial x^*(c)}{\partial c} > 0$ *and* $x^{**}(c_1)$ *when* $c \in \left[\bar{c}, \bar{\bar{c}}\right]$ *with* $\frac{\partial x^{**}(c)}{\partial c} < 0$ *such that Blue sets*

$$s_1 = \begin{cases} b \text{ when } c < \underline{c} \\ \begin{cases} b \text{ if } x \geq x^*(c) \\ s_0 \text{ if } x < x^*(c) \end{cases} \text{when } c \in [\underline{c}, \bar{c}] \\ \begin{cases} b \text{ if } x \leq x^{**}(c) \\ s_0 \text{ if } x > x^{**}(c) \end{cases} \text{when } c \in \left[\bar{c}, \bar{\bar{c}}\right] \\ s_0 \text{ when } c > \bar{\bar{c}} \end{cases}$$

iii. *When* $b < r < s_0$ *then Blue sets*

$$s_1 = \begin{cases} b \text{ when } c \leq (1+\delta)\Delta_b(s_0) \\ s_0 \text{ when } c > (1+\delta)\Delta_b(s_0) \end{cases}$$

Proof: See the Appendix.

Although implicit in the previous discussion, it is worth emphasizing two properties that follow from the Proposition. First, there is an asymmetry in the strategy followed by the first period majority coalition when the standard is located to its left or to its right. For Blue it is not the same to face an initial standard that is too conservative or too liberal, even when both coalitions agree in that perception. That emphasizes the role played by the initial standard. Second the distances between the initial standard and the ideal points (either b or r) are not sufficient to characterize the strategy followed by the first period majority. Blue follows different strategies when the initial standard is located to the left or to the right even when the distance to its ideal point is the same. That emphasizes the role played by the distribution of the coalitions ideal points relative to the initial standard. As we will discuss in Section 4, these two properties do not follow from the restriction that the first period majority can only choose between the initial standard and optimal points.

4.2.MAXIMUM STABILITY WITHIN A 3-PERIOD MODEL

In this Section, we derive the conditions under which opposed coalitions never change the standard (*maximum stability*) in the context of a three period model and the initial standard is very close to Blue's optimal ideology.[15] We conjecture that the set of costs that support *maximum stability* goes down with the number of periods and converges to the set of costs that is greater than a certain threshold independent of the value of x. More specifically, we suggest that there exists a set of costs in which *maximum stability* holds only when x takes intermediate values and that region tends to disappear when the number of periods gets very

[15] Note that that does not include scenarios in which there is a positive probability that one of the coalitions changes the standard at any period within the T-periods.

large. That is, when the cost of a change takes intermediate values, we know that neither Blue nor Red change the standard when this is equal to the original value. Indeed Blue follows a strategy that changes the standard when x is large enough and Red follows a strategy that changes the standard when x is small enough because both expect to preserve that standard in the future. However, when x takes intermediate values then no coalition is willing to face the risk that the opposite coalition, in the future, will change the standard to its extreme preferred ideology. The result does not hold if the cost is small, because the majority coalition would be willing to face the cost of changing the standard back in the future if the opposite coalition changes it in the future.

Proposition 3, which we formally prove in the Appendix, shows the region of *maximum stability* within the 3-Periods model. Figure 3 identifies those regions graphically.

<<Insert Figure 3 about here>>

PROPOSITION 3 (*Maximum Stability* in a 3-Period Model):

The standard never deviates from s_0 when

i. $c \in [\Delta_r(s_0), \Delta]$ *and* $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta(1+\delta)}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta(1+\delta)}}\right) b\right]$ *for all* $x > \underline{x}(c)$.

ii. $c \in [\Delta_r(s_0), \Delta]$ *and* $s_0 \in \left[\frac{1}{1+\sqrt{1+\delta(1+\delta)}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta(1+\delta)}}\right) b, \frac{1}{1+\sqrt{1+\delta}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$ *for all* $x \in [\underline{x}(c), \overline{x}(c)]$.

iii. $c > \Delta$ *for all values of* x.

Proof: See the previous discussion.

The proposition tells us that, in a three-period model, *maximum stability* takes place as long as the interaction between costs and the likelihood of keeping a majority deter change. Trivially, as long as costs are too high, a standard never changes no matter the number of periods. However, when costs are reasonable (but not too low, otherwise coalitions change any standard as soon as they have an opportunity), *maximum stability* is still a possibility because of the way one coalition would react to the standard set by the previous majority. More central for us, the conditions for *maximum stability* are stricter in the three period than in the two period model suggesting that *maximum stability* becomes less common the more periods the coalitions take into account when they make their decisions.

4.3. GENERAL RESULTS

Here we relax the assumption that $s_t \in \{s_0, b, r\}$ and re-calculate Blue and Red optimal strategies allowing for $s_t \in [0,1]$. As we did in the discrete model ($s_t \in \{s_0, b, r\}$), in the continuous model ($s_t \in [0,1]$) we find that Blue's strategy centrally depends on whether the coalitions have opposed or aligned ideologies, where the initial standard is located at and whether the cost of a change takes extreme or intermediate values.

Once more we find that the strategy followed by Blue is independent of x when the initial standard is extreme conservative (Blue and Red are aligned at the left wing). We also find that Blue sets a standard that weakly decreases with x when coalitions have opposed ideologies and the cost of a change takes intermediate values.[16]

[16] That is, the more likely is that Blue remains as the majority coalition in the second period, the weakly closer to b is the standard set by Blue in the first period.

The main difference with the discrete model is that, in the continuous model, Blue´s decisions to be “aggressive” or to “accommodate” have a richer meaning than the one it has in the discrete model. While in the context of the discrete model Blue is “aggressive” when it sets b and accommodating when it keeps s_0, in the context of the continuous model Blue might be “aggressive”, meaning that it sets b, “semi-accommodating”, it sets $r-\sqrt{c}\in[b,s_0]$, “accommodating”, it keeps s_0, or “concessive”, it sets $r-\sqrt{c}\in[s_0,r]$. Standard $r-\sqrt{c}$ is the most liberal value that convinces second period Red majority not to set standard r. Beyond the specifics, the strategy followed by Blue tells us that, for opposed coalitions, *the more likely is that the Court is stable (same coalition is in majority in both periods) the closer to the ideology of the majority coalition is the standard set by that coalition in the first period.*

Propositions 1G shows that when c takes intermediate values, Blue’s disposition to move the standard closer to b increases with x.

PROPOSITION 1G (Blue’s decisions at t = 1 with opposed coalitions):

When $s_0\in[b,r]$ *there exists* $\underline{c}$, $\overline{c}$ *and* $x^*(c;s_0), x_1^*(c;s_0), x_2^*(c;s_0)$ *when* $c\in[\underline{c},\overline{c}]$ *with* $\frac{\partial x^*(c;s_0)}{\partial c}>0$ *and* $x_1^*(c;s_0), x_2^*(c;s_0)$ *concave functions such that Blue sets*

$$s_1=\begin{cases} b \text{ when } c<\underline{c} \\ \begin{cases} b \text{ if } x>\max\{x^*(c;s_0),x_2^*(c;s_0)\} \\ s_0 \text{ if } x\in\begin{bmatrix}\min\{x_1^*(c;s_0),x_2^*(c;s_0)\}, \\ \max\{x^*(c;s_0),x_2^*(c;s_0)\}\end{bmatrix} \\ r-\sqrt{c} \text{ if } x<\min\{x_1^*(c;s_0),x_2^*(c;s_0)\}\end{cases} \text{ when } c\in[\underline{c},\overline{c}] \\ s_0 \text{ when } c>\overline{c}\end{cases}$$

Proof: See the next discussion.

As a starting point we note that in the first period Blue optimally fixes a standard that belongs to the set $\{b, r-\sqrt{c}, s_0\}$. In the continuous model $s_1 = r-\sqrt{c}$ might be optimal because there are scenarios in which simultaneously two statements are true. First, if Blue sets $s_1 = b$ and in response a second period Red majority sets $s_2 = r$ then Blue's payoff from facing a more liberal standard than $r-\sqrt{c}$ in the first period (which is b) is smaller than Blue's cost from facing a more conservative standard than $r-\sqrt{c}$ in the second period (which is r) and ergo Blue prefers $r-\sqrt{c}$ to b. Second, if Blue keeps $s_1 = s_0$ and c is small enough then Blue's cost from facing a more conservative standard than $r-\sqrt{c}$ (which is s_0) is greater than Blue's cost from changing the initial standard and ergo prefers $r-\sqrt{c}$ to s_0.[17]

Evidently $s_1 = r-\sqrt{c}$ is derived from

$$1-c = 1-(r-s_1)^2 \rightarrow s_1 = r-\sqrt{c}$$

Note that $r-\sqrt{c} < s_0$ when $c > \Delta_r(s_0)$ but $r-\sqrt{c} > s_0$ when $c < \Delta_r(s_0)$.

Blue's optimal strategy in the first period follows after we compare Blue's payoffs when it decides $b,\ r-\sqrt{c}$ or s_0. While in Section 3.1 we wrote general expressions for $U_B(b)$ and $U_B(s_0)$ here we write a general expression for $U_B(r-\sqrt{c})$.

$$U_B(r-\sqrt{c}) = \begin{cases} 1-\Delta_b(r-\sqrt{c}) - c + \delta\left((1-c)x + (1-x)\left(1-\Delta_b(r-\sqrt{c})\right)\right) \text{ if } c < \dfrac{\Delta}{4} \\ \left(1-\Delta_b(r-\sqrt{c})\right)(1+\delta) - c \quad \text{if } c \geq \dfrac{\Delta}{4} \end{cases}$$

[17] Blue prefers at least one of $\{b, r-\sqrt{c}, s_0\}$ to any other value of s_1. To see that, consider that $b < r-\sqrt{c} < s_0$, then all $s_1 \in [b, r-\sqrt{c}]$ are at least dominated by b because we are in a region in which a second period Red majority changes the standard, hence Blue gets a maximum payoff in the first period and as a second period majority if the standard is b and not s_1. In addition, all $s_1 \in [r-\sqrt{c}, s_0]$ are at least dominated by $r-\sqrt{c}$ because we are in a region in which a second period Red majority does not change the standard, hence Blue gets a maximum payoff in the first period and as a second period majority if the standard is $r-\sqrt{c}$ and not s_1. The same steps can be used to prove that all $s_1 \notin \{b, r-\sqrt{c}, s_0\}$ are dominated when $b < s_0 < r-\sqrt{c}$.

If Blue sets $r-\sqrt{c}$ in the first period then in the second period a Blue majority changes it to b if $c<\Delta/4$ but keeps it otherwise. Evidently a Red majority does not change $r-\sqrt{c}$ as this standard makes Red indifferent to change it to r. Unlike $U_B(b)$ and $U_B(s_0)$ this function is everywhere continuous but like $U_B(b)$ it does not depend on s_0.

We first show that there always exist $\underline{c}$ and $\overline{c}$ such that, for all values of x, Blue changes the standard to b when $c<\underline{c}$ and Blue keeps the standard as s_0 when $c>\overline{c}$. Later we discuss Blue´s strategy when $c\in[\underline{c},\overline{c}]$.

From Proposition 1 we know that when coalitions have opposed ideologies then Blue prefers $s_1=b$ to $s_1=s_0$ for all values of x when the cost is either smaller than $\Delta_b(s_0)$ (if $s_0>\frac{r+b}{2}$) or smaller than $(1+\delta)\Delta_b(s_0)-\delta\Delta$ (if $s_0<\frac{r+b}{2}$). In addition, when $c<\Delta/4$ Blue prefers $s_1=b$ to $s_1=r-\sqrt{c}$ when

$$U_B(b)>U_B\left(r-\sqrt{c}\right)\leftrightarrow\Delta_b\left(r-\sqrt{c}\right)+\delta cx>\delta(1-x)\left(\Delta-\Delta_b\left(r-\sqrt{c}\right)\right)\quad(8)$$

The intuition behind (8) is that Blue prefers b when the disutility for facing a more conservative standard at t = 1 ($r-\sqrt{c}$ instead of b) added with the cost of changing that standard to b at t = 2 is greater than the benefit from preventing Red to change the standard at t = 2. This inequality holds for all values of x for a given c when $x=0$.[18] That is

$$\Delta_b\left(r-\sqrt{c}\right)>\frac{\delta}{1+\delta}\ \Delta\leftrightarrow c<\Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2$$

We conclude that

[18] Clearly $\left(c-\Delta_b\left(r-\sqrt{c}\right)+\Delta\right)$ is positive because $\Delta>\Delta_b\left(r-\sqrt{c}\right)$.

$$\underline{c} = \begin{cases} \min\left\{\Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2, (1+\delta)\Delta_b(s_0)-\delta\Delta\right\} \text{ if } s_0 \in \left[\frac{r+b}{2}, r\right] \\ \min\left\{\Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2, \Delta_b(s_0)\right\} \text{ if } s_0 \in \left[b, \frac{r+b}{2}\right] \end{cases}$$

On the other extreme of values of c, we know from Proposition 1 that when $c > (1+\delta)\Delta_b(s_0)$ Blue prefers to keep s_0 instead to set b for all values of x. In addition, after we compare $U_B(s_0)$ with $U_B(r-\sqrt{c})$ it follows that Blue always keeps s_0 when

$$U_B(s_0) > U_B(r-\sqrt{c}) \leftrightarrow (1-\Delta_b(s_0))(1+\delta) > \left(1-\Delta_b(r-\sqrt{c})\right)(1+\delta) - c$$

$$\leftrightarrow c > (1+\delta)\Delta_b(s_0) - (1+\delta)\Delta_b(r-\sqrt{c}) \qquad (5)$$

Which is evidently true when $c > (1+\delta)\Delta_b(s_0)$ in case that $(1+\delta)\Delta_b(s_0) > \Delta_r(s_0)$ and also evidently true when $c > \Delta_r(s_0)$, in case that $\Delta_r(s_0) > (1+\delta)\Delta_b(s_0)$. We conclude that

$$\overline{c} = \begin{cases} \Delta_r(s_0) \text{ if } s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta}} r + \left(1-\frac{1}{1+\sqrt{1+\delta}}\right) b\right] \\ (1+\delta)\Delta_b(s_0) \text{ if } s_0 \in \left[\frac{1}{1+\sqrt{1+\delta}} r + \left(1-\frac{1}{1+\sqrt{1+\delta}}\right) b, r\right] \end{cases}$$

At this point we are only missing the strategy followed by Blue in the first period when c takes intermediate values ($c \in [\underline{c}, \overline{c}]$). Once more we have to distinguish scenarios in which s_0 is closer to b or closer to r. As we did with the discrete model, we impose $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta}} r + \left(1-\frac{1}{1+\sqrt{1+\delta}}\right) b\right]$, the solution for the rest of the cases is analogous.

We show that there exists a set of thresholds in the probability that Blue keeps its majority condition in the second period that substantially changes Blue's strategy in the first period. If x is larger than any of those thresholds then Blue moves the standard closer to its preferred ideology when compared to what Blue decides when x is lower than the threshold.

That either means that Blue aggressively moves the standard instead to (semi-) accommodate the standard or means that Blue accommodate the standard instead to move it closer to the Red's ideology. Parameter s_0 determines which case we are on.

Because $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$ then $\overline{c} = \Delta_r(s_0)$ but $\underline{c}$ is either equal to $= \Delta\left(1 - \sqrt{\frac{\delta}{1+\delta}}\right)^2$ or equal to $\Delta_b(s_0)$. Next we discuss these two possibilities separately.

If s_0 is close enough to b then $\underline{c} = \Delta_b(s_0)$. That implies that if Blue prefers b to s_0, it also prefers b to $r - \sqrt{c}$. Indeed, when $c \in [\Delta_b(s_0), \Delta_r(s_0)]$ these two preferences are

$$x > x^*(c) = \frac{c - \Delta_b(s_0)}{\delta \Delta_b(s_0)} \quad \text{(Blue prefers } b \text{ to } s_0\text{)}$$

$$x > x_2^*(c) = \begin{cases} 1 + \frac{1}{\delta}\left(1 - \frac{c(1+\delta) + \Delta}{2\sqrt{c\Delta}}\right) \text{ if } c \in \left[\Delta_b(s_0), \frac{\Delta}{4}\right] \\ 1 - \frac{1+\delta}{\delta}\left(1 - \sqrt{\frac{c}{\Delta}}\right)^2 \text{ if } c \in \left[\frac{\Delta}{4}, \Delta_r(s_0)\right] \end{cases} \text{(Blue prefers } b \text{ to } r - \sqrt{c}\text{)}$$

In which $x^*(c) > x_2^*(c)$.[19] To better grasp the intuition behind this inequality we re-write Blue pay-offs when it sets b and $r - \sqrt{c}$ respectively

$$U_B(b) = 1 - c + \delta\big(x + (1-x)(1-\Delta)\big)$$
$$U_B\big(r - \sqrt{c}\big) = 1 - \Delta_b\big(r - \sqrt{c}\big) - c + \delta\left((1-c)x + (1-x)\left(1 - \Delta_b\big(r - \sqrt{c}\big)\right)\right)$$

Although hard to see at first, Blue prefers b to $r - \sqrt{c}$ only if the cost is small enough. Blue prefers the first than the second when the cost of having a more conservative standard at t =

[19] $x_2^*(c)$ is a concave function in c with a slope smaller than $1/\delta\Delta_b(s_0)$ for all $c \in [\Delta_b(s_0), \Delta_r(s_0)]$. It is enough to prove that $\left.\frac{\partial\left(1+\frac{1}{\delta}\left(1-\frac{c(1+\delta)+\Delta}{2\sqrt{c\Delta}}\right)\right)}{\partial c}\right|_{c=\Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2} < \frac{1}{\delta\Delta_b(s_0)}$. But that is equivalent to $\Delta_b(s_0) < \Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^{\frac{3}{2}} /(1 - (1+\delta)\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2)$ which is true for all δ when $\Delta_b(s_0) < \Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2$.

1 is large ($\Delta_b\big(r-\sqrt{c}\big)$), the cost for setting a new standard at t = 2 is also large (δcx) and the cost for Red setting *r* instead of keeping $r-\sqrt{c}$ is small ($\delta(1-x)\Big(\Delta-\Delta_b\big(r-\sqrt{c}\big)\Big)$). The key point here is to note that $\Delta_b\big(r-\sqrt{c}\big)=(r-\sqrt{c}-b)^2$ is decreasing in *c*. Hence if c goes down then $\Delta_b\big(r-\sqrt{c}\big)$ goes up and $\Delta-\Delta_b\big(r-\sqrt{c}\big)$ goes down. It is true that δcx also goes down but when $c\in[\Delta_b(s_0),\Delta_r(s_0)]$, the two first effects dominate the last one.

Hence Blue prefers to set b than the other two options when $c<(1+\delta x)\Delta_b(s_0)$. On the other side, we know that Blue prefers s_0 to b when $c\geq(1+\delta x)\Delta_b(s_0)$. The question is whether that coalition prefers s_0 to $r-\sqrt{c}$ as well. This time the relevant comparison is

$$U_B(s_0)=1-\Delta_b(s_0)+\delta\big(x(1-\Delta_b(s_0))+(1-x)(1-\Delta)\big)$$

with

$$U_B\big(r-\sqrt{c}\big)=\begin{cases}1-\Delta_b\big(r-\sqrt{c}\big)-c+\delta\Big((1-c)x+(1-x)\Big(1-\Delta_b\big(r-\sqrt{c}\big)\Big)\Big)\text{ if } c<\dfrac{\Delta}{4}\\ \Big(1-\Delta_b\big(r-\sqrt{c}\big)\Big)(1+\delta)-c\quad\text{if } c\geq\dfrac{\Delta}{4}\end{cases}$$

Which defines the following concave function.

$$x>x_1^*(c)=\begin{cases}\dfrac{\Delta_b(s_0)+\delta\Delta-(1+\delta)\Delta_b\big(r-\sqrt{c}\big)-c}{\delta\Big(\Delta-\Delta_b(s_0)+c-\Delta_b\big(r-\sqrt{c}\big)\Big)}\text{ if } c<\dfrac{\Delta}{4}\\ \dfrac{\Delta_b(s_0)+\delta\Delta-(1+\delta)\Delta_b\big(r-\sqrt{c}\big)-c}{\delta\big(\Delta-\Delta_b(s_0)\big)}\text{ if } c\geq\dfrac{\Delta}{4}\end{cases}\text{(Blue prefers } s_0 \text{ to } r-\sqrt{c}\text{)}$$

For example when $c\geq\Delta/4$, Blue keeps the standard when the savings of an initial change added to the benefit of keeping a more liberal standard both at t = 1 and at t = 2 with a Blue majority are greater than the cost of facing a more conservative standard if Red becomes majority at t = 2. It follows that the only cost faced by Blue when keeping s_0 goes down when x goes up and explains the existence of threshold $x_1^*(c)$.

Knowing that a contradiction argument implies that $x_1^*(c) > x_2^*(c)$, in Figure T.1 we show the unique graphical representation of these thresholds when s_0 is very small. [20]

<<Insert Figure T.1 about here>>

The analysis implies that $\max\{x^*(c; s_0), x_2^*(c; s_0)\} = x^*(c; s_0)$ as well as $\min\{x_1^*(c; s_0), x_2^*(c; s_0)\} = x_1^*(c; s_0)$ which by itself implies that $x_2^*(c)$ is irrelevant. Hence, we can summarize Blue's strategy when $\underline{c} = \Delta_b(s_0)$ for all values of c only conditional on $x^*(c; s_0)$ and $x_1^*(c; s_0)$ as it is done in Figure 2.1.A. That is, when the cost of a change is moderately large (large-intermediate) Blue keeps the initial standard when x is large but moves it closer to r otherwise. Instead when the cost of a change is moderately small (small-intermediate) Blue sets b when x is large, keeps the initial standard when x takes intermediate values but moves the standard closer to r when x is small.

<<Insert Figure 2.1.A about here>>

It might be counter intuitive that when $c \in [(1+\delta)\Delta_b(s_0), \Delta_r(s_0)]$ Blue keeps the standard when x is large but changes it when x is small. The answer is given by the fact that $r - \sqrt{c}$ is located to the right of s_0! That is, Blue's "aggressive" decision is to keep the standard while Blue's "accommodating" decision is to move it closer to r.

The same previous mechanics can be applied to discuss the case in which s_0 is close enough to $\frac{1}{1+\sqrt{1+\delta}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b$. Under that condition, $\underline{c} = \Delta\left(1 - \sqrt{\frac{\delta}{1+\delta}}\right)^2 < \Delta_b(s_0)$ which implies that there exist a cost, that we denote $\hat{c} \in [\Delta_b(s_0), (1+\delta x)\Delta_b(s_0)]$, at which

[20] If $x_2^*(c) > x_1^*(c)$ then there exists a range of values of c in which $x^*(c) > x_2^*(c) > x_1^*(c)$ which implies that when $x \in [x_1^*(c), x_2^*(c)]$ then s_0 is preferred to b and b is preferred to $r - \sqrt{c}$ and $r - \sqrt{c}$ is preferred to s_0 which is contradiction.

$x^*(c)$ and $x_2^*(c)$ intersect. Indeed, $\hat{c}$ is the cost that makes Blue indifferent between the three decisions $\{b, r-\sqrt{c}, s_0\}$.[21] Figure T.2 shows the unique distribution for the three thresholds.

<<Insert Figure T.2 about here>>

The figure implies that in addition to regions of values of c in which 1) Blue sets b when x is large, keeps the initial standard when x takes intermediate values but moves the standard closer to r when x is small ($c \in [\hat{c}, (1+\delta)\Delta_b(s_0)]$) and 2) Blue keeps s_0 when x is large but moves the standard closer to r otherwise ($c \in [(1+\delta)\Delta_b(s_0), \Delta_r(s_0)]$) there exists a region, absent in Figure 2.1.A, in which Blue sets b when x is large but sets $r-\sqrt{c}$ otherwise ($c \in [\underline{c}, \hat{c}]$). In other words, this time

$$\max\{x^*(c;s_0), x_2^*(c;s_0)\} = \begin{cases} x_2^*(c;s_0) \text{ when } c < \hat{c} \\ x^*(c;s_0) \text{ when } c > \hat{c} \end{cases}$$

$$\min\{x_1^*(c;s_0), x_2^*(c;s_0)\} = x_1^*(c;s_0) \text{ for all values of } c.$$

<<Insert Figure 2.1.B about here>>

Figure 2.1.B. summarizes Blue strategies when $\underline{c} = \Delta\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)^2$ conditional on the values of c. *Overall, both figures show that when c takes intermediate values, the probability with which Blue sets a standard closer to b increases with x.*

5. DISCUSSION

The main insight from our model is that uncertainty about future court preferences might deter legal change in the present. A court dominated by the same party leaning produces

[21] If Blue is indifferent between b and s_0 and between b and $r-\sqrt{c}$ then it is also indifferent between s_0 and $r-\sqrt{c}$.

stable law in the sense that the prevalent majority pushes their standard when costs are minimal and there is no need for future change (assuming everything else stays the same). However, in a court where majoritarian coalitions alternate, we could also have stable law in the sense that the initial standard is unchanged because each side is deterred by the threat of triggering a costly "standard-race" (technically, notice that, since we work with a reduced form model, standards are unchanged because the first majority has strategically set out the standard such that the rival group does not find it worthwhile to change it). The opposite situation, legal instability, takes place when majoritarian coalitions endure or the costs of adjustment are sufficiently low to make the possible "standard-race" a reasonable risk to take (in our reduced form model, we have a "standard-race" when Blue picks *b* in period one and Red picks *r* in period two).

We suggest that this insight has possible policy applications. The canonical parameters are the expectations coalitions have concerning the future and the relative costs of adjustment, both inevitable tainted with hindsight biases when we discuss specific illustrations. Still, one general application is explaining why increasing court polarization (Bartels, 2015; Clark, 2008; Epstein et al, 2007; Gooch, 2015) may not bring legal change. . If polarization is associated with increasing the relative costs borne by each side when a "standard-race" takes place, then it actually enhances rather than diminish legal stability. In fact, observed ideological swings, for example, from a "liberal court" (Warren court; Powe, 2002) to more "conservative courts" (Burger and Rehnquist courts; Lindquist and Solberg, 2007) might not produce as much as change as the attitudinal model would suggest precisely because of deterrence induced by the possibility of triggering a costly instability. Yet, once a new ideological majority has a perception of significant durability, we should expect legal change to be observed which is consistent with long periods of a "liberal court" or a "conservative court" (Tushnet, 2005).[22] Similarly, evidence about precedent in the US Supreme Court seems to support our modeling approach - as predicted by our model, less radical precedents are more likely to survive.[23]

Turning to anecdotal evidence and historical debates, our model might provide an additional reasoning to the opposing accounts by Leuchtenburg (1995) and Cushman (1998) concerning the constitutional change in the 1930s (New Deal) – an internal threat of costly instability provides the analytical framework for a continuous gradual change rather than a full switch story.

An additional angle of analysis is the prevalence of confrontational mood versus more harmonious working environment. For example, Crawford Greenburg (2007) provides two useful examples – Lee v. Weisman (1992) and Lawrence v. Texas (2003). Justice Kennedy wrote the majority opinion. The author explains how he had to ponder changing precedent - in the latter case, Bowers v. Hardwick (1986)- and find the appropriate language to search for compromise as much as possible in order to avoid turmoil in the future. Although, in both cases, Justice Kennedy could not ultimately avoid a split, he tried to suppress hostile language while engaging with the (minority) conservative wing of the court (Crawford Greenberg,

[22] A few critics of the modern trends in the US Court have voiced that confrontational polarization changes the law too far and too often, thus documenting that a "standard-race" is a real possibility. For example, McCloskey (2010) argues that when pragmatic compromise is replaced by "negative capability", the resistance to push a logical extreme is broken and legal standards become uncertain. According to the author, this happened in the New Deal period as well as in the "liberal court" of the 1950s and 1960s because "judges of the Supreme Court itself did not know the history of the bench they occupied, or had failed to understand it".

[23] In their study of 6,363 precedents, from 1946 to 2001, Hansford and Spriggs (2006) find the survival of a precedent seems to be helped by ideological proximity and vitality (as measured by more prior positive citations and less negative citations). As vitality increases, ideological proximity matters more. However, focusing on majority interpretation of law, the same authors conclude that only ideological distance seems to matter (the majority seems to use precedents closer to the median ideology) while vitality seems unimportant.

2007, at 36-63 and at 139-163). Eventually both cases set new precedents that have largely been stable.[24]

Our model could also be understood as explaining why minimalism in constitutional review is likely to prevails over fundamentalism or extremism (visionaries), using the language of Sunstein (2005, 2009). Minimalism (either conservative or liberal) is defined as proceeding step by step without unbalancing the law, preserving the past, with slow change and no legal revolution in opposition to visionaries (either conservative or liberal) who seek big and sudden change (either back to the original meaning or moving forward in the progressive agenda). In fact, our model explains why minimalism might emerge even when both sides are polarized by visionaries. In a way, visionaries are afraid of each other and might converge on minimalism as the best possible solution to legal debates.

In Europe, the relationship between constitutional courts and supreme courts has not been uniform. Some countries exhibit a tense relationship (Italy and Spain quite remarkably, France more recently) whereas other countries have a reasonably peaceful engagement (Germany comes to mind). Different incentives might explain these differences (Garoupa and Ginsburg, 2015). Our results provide an additional useful reasoning – we could imagine that the situation is similar to our model where both "majority" (constitutional court) and "minority" (supreme court) know their relative strength in the future (constitutional court always prevails over supreme court). However, in countries with more unstable composition (such as France, Italy or Spain), the preference of the constitutional court could change in

[24] Another author, looking at different personalities and temperaments within the same ideological area (Rosen, 2007), argues that justices more inclined to compromise or less confrontational might have more influence in the US Supreme Court due to their ability to build stable coalitions. Specifically, on the liberal side, he compares Black and Douglas; on the conservative side, he compares Rehnquist and Scalia.

time, therefore triggering more conflicts with the “minority”. In countries with more stable composition (such as Germany, Austria or Portugal), the preferences of the constitutional court are less likely to change in time, hence allowing some form of settlement with the “minority” or an inclination for gradual change.

6. CONCLUSIONS

In this article, we presented a discussion of stability of the law based on the game played by coalitions in a court of law. Changing the law has two aspects. There is a direct cost from setting a new standard, therefore, limiting legal volatility to a certain degree. However, there is an indirect cost – a change of law in present could trigger a reaction from the opposing coalition in the future. Therefore, a current majority could be deterred from shaping the law more actively if the future is too uncertain.

In courts where composition is stable and the dynamics of majority and minority is largely unchanged, we expect the majority to adjust the law to their preferences at some point. However, a court with a strong and uncertain dynamics of majority and minority can produce two very different results. If the direct costs of changing the law are somehow relevant, the coalitions may settle on not reforming the law and we have *maximum stability*. In this case, judicial polarization, for example, is consistent with stable law. However, when the direct costs of changing the law are moot, we may observe huge swings in the law reflecting *maximum instability*.

Our article points out that the relationship between court composition and legal stability could be more nuanced than anticipated. Important ideological shifts in composition could have less impact in the law precisely because the different coalitions are deterred by the possibility of triggering a costly "standard-race" that harms both sides with excessive costs.

REFERENCES

Baker, Scott and Claudio Mezzetti. 2012. A Theory of Rational Jurisprudence, Journal of Political Economy 120: 513-551.

Bartels, Brandon L. 2015. The Sources and Consequences of Polarization in the US Supreme Court, in American Gridlock: The Sources, Character, and Impact of Political Polarization (James A. Thurber and Antoine Yoshinaka, eds), New York, NY: Cambridge University Press.

Bergara, Mario, Barak D. Richman and Pablo T. Spiller. 2003. Modeling Supreme Court Strategic Decision Making: The Congressional Constraint, Legislative Studies Quarterly 28 (2): 247-280.

Bustos, Álvaro and Tonja Jacobi. 2014. Strategic Judicial Preference Revelation, Journal of Law and Economics, 57 (1): 113-137.

Bustos, Álvaro and Tonja Jacobi. 2019. Judicial Choice among Cases for Certiorari, Supreme Court Economic Review, forthcoming,

Callander, Steven and Tom S. Clark. 2017. Precedent and Doctrine in a Complicated World, American Political Science Review 111(1):184-203.

Cameron, Charles M. and Lewis A. Kornhauser. 2010. Modeling Collegial Courts III: Adjudication Equilibria, https://papers.ssrn.com/sol3/papers.cfm?abstract_id=2153785

Cameron, Charles M. and Lewis A. Kornhauser. 2017. Theorizing the U.S. Supreme Court, Oxford Research Encyclopedia of Politics.

Cameron, Charles M., Lewis A. Kornhauser and Giri Parameswaran. 2019. Stare Decisis and Judicial Log-Rolls: A Gains-from-Trade Model, Rand Economic Journal, forthcoming.

Caminker, Evan H. 1999. Sincere and Strategic Voting Norms on Multimember Courts, Michigan Law Review 97(8): 2297-2380.

Clark, Tom S. 2008. Measuring Ideological Polarization on the United States Supreme Court, Political Research Quarterly 62 (1): 146-157.

Crawford Greenburg, Jan. 2007. Supreme Conflict: The Inside Story of the Struggle for Control of the United States Supreme Court, NY: Penguin Group.

Cushman, Barry. 1998. Rethinking the New Deal Court: The Structure of a Constitutional Revolution. New York, NY: Oxford University Press.

Epstein, Lee and Tonja Jacobi. 2010. The Strategic Analysis of Judicial Decisions, Annual Review of Law and Social Science 6: 341–58.

Epstein, Lee and Jack Knight. 1998. The Choices Justices Make. Washington, D.C.: C.Q. Press.

Epstein, Lee, William M. Landes, and Richard A. Posner. 2013. The Behavior of Federal Judges: A Theoretical and Empirical Study of Rational Choice. Boston: Harvard University Press.

Epstein, Lee, Andrew D. Martin, Kevin M. Quinn, and Jeffrey A. Segal. 2007. Ideological Drift among Supreme Court Justices: Who, When, and How Important, Northwestern University Law Review 101: 1483-1541.

Garoupa, Nuno and Tom Ginsburg. 2015. Judicial Reputation: A Comparative Theory. Chicago, Illinois: University of Chicago Press.

Gennaioli, Nicola and Andrei Shleifer. 2007. The Evolution of Common Law, Journal of Political Economy 115: 43-68.

Gely, Rafael and Pablo T. Spiller. 1990. A Rational Choice Theory of Supreme Court Statutory Decisions with Applications to the State Farm and Grove City Cases, Journal of Law, Economics, and Organization 6:263-300.

Gely, Rafael and Pablo T. Spiller. 1992. The Political Economy of Supreme Court Constitutional Decisions: The Case of Roosevelt's Court-Packing Plan, International Review of Law and Economics 12(1):45-67.

González Bertomeu, Juan, Lucia Dalla Pellegrina, and Nuno Garoupa. 2016. Estimating Judicial Ideal Points in Latin America: The Case of Argentina, Review of Law and Economics 13, https://doi.org/10.1515/rle-2015-0040.

Gooch, Donald M. 2015. Ideological Polarization on the Supreme Court: Trends in the Court's Institutional Environment and Across Regimes, 1937-2008, American Politics Research 43, 999-1040

Hansford, Thomas G. and James F. Spriggs II. 2006. The Politics of Precedent on the US Supreme Court, Princeton NJ: Princeton University Press.

Hanssen, F. Andrew Hanssen. 2004. There a Politically Optimal Level of Judicial Independence?, American Economic Review 94, 712-729.

Helmke, Gretchen. 2005. Courts Under Constraints: Judges, Generals, and Presidents in Argentina, New York: Cambridge University Press.

Helmke, Gretchen and Frances Rosenbluth. 2009. Regimes and the Rule of Law: Judicial Independence in Comparative Perspective, Annual Review of Political Science 12, 345-366.

Kornhauser, Lewis A. 1989. An Economic Perspective on Stare Decisis, Chicago-Kent Law Review 65, 63-92.

Kornhauser, Lewis A. 1992a. Modeling Collegial Courts I: Path-Dependence, International Review of Law and Economics 12(2):169-185.

Kornhauser, Lewis A. 1992b. Modeling Collegial Courts. II. Legal Doctrine, Journal of Law, Economics, and Organization 8(3):441-470.

Kornhauser, Lewis A. 1995. Adjudication by a Resource-Constrained Team: Hierarchy and Precedent in a Judicial System, Southern California Law Review 68:1605-1629.

Landes, William M. and Richard A. Posner. 1975. The Independent Judiciary in an Interest-Group Perspective, Journal of Law and Economics 18: 875-901.

La Porta, Rafael, Florencio Lopez-de-Silanes and Andrei Shleifer. 2008. The Economic Consequences of Legal Origins, Journal of Economic Literature 46 (2): 285-322.

Leuchtenburg, William E. 1995. The Supreme Court Reborn: The Constitutional Revolution in the Age of Roosevelt, New York, NY: Oxford University Press.

Lindquist, S. A. and R. S. Solberg. 2007. Judicial Review by the Burger and Rehnquist Courts: Explaining Justices' Responses to Constitutional Challenges, Political Research Quarterly 60, 71-90.

Maltzman, Forrest, James F. Spriggs II, and Paul J. Wahlbeck. 2000. Crafting Law on the Supreme Court: The Collegial Game. New York: Cambridge University Press.

Martin, Andrew D., Kevin M. Quinn, and Lee Epstein. 2005. The Median Justice on the United States Supreme Court, North Carolina Law Review 83: 1275-1322.

McCloskey, Robert G. 2010. The American Supreme Court, 5th edition, Chicago: University of Chicago Press.

Moraski, Bryon and Charles Shipan. 1999. The Politics of Supreme Court Nominations: A Theory of Institutional Constraints and Choices, American Journal of Political Science 43 (4): 1069-1095.

Niblett, Anthony, Richard A. Posner, and Andrei Shleifer. 2010. The Evolution of a Legal Rule, Journal of Legal Studies 39: 325-358.

O'Hara, Erin. 1993. Social Constraint or Implicit Collusion? -Toward a Game Theoretic Analysis, Seton Hall Law Review 24, 736-778.

Oliveira, Maria Angela and Nuno Garoupa. 2011. Choosing Judges in Brazil: Reassessing Legal Transplants from the United States, American Journal of Comparative Law 59 (2): 529-561.

Parameswaran, Giri. 2018. Endogenous Cases and the Evolution of the Common Law, Rand Journal of Economics 49, 781-818.

Ponzetto, Giacomo A. M. and Patricio A. Fernandez. 2008. Case Law versus Statute Law: An Evolutionary Comparison, Journal of Legal Studies 37: 379-430.

Posner, Richard A. 1993. What Do Judges and Justices Maximize? (The Same Thing Everybody Else Does), Supreme Court Economic Review 3(1):1-41.

Posner, Richard A. 2004. Judicial Behavior and Performance: An Economic Approach, Florida State University Law Review 32: 1259-1279.

Posner, Richard A. 2010. How Judges Think. Cambridge, Mass: Harvard Univ. Press.

Powe. Lucas A. 2002. The Warren Court and American Politics. Cambridge, Mass: Belknap Press.

Ramseyer, J. Mark and Eric B. Rasmusen. 1997. Judicial Independence in a Civil Law Regime: The Evidence from Japan, Journal of Law, Economics and Organization 13: 259-286.

Rasmusen, Eric B. 1994. Judicial Legitimacy as a Repeated Game, Journal of Law, Economics and Organization 10, 63-83.

Rosen, Jeffrey. 2007. The Supreme Court: The Personalities and Rivalries that Defined America, NY: Holt.

Segal, Jeffrey A. 1997. Separation-of-Powers Games in the Positive Theory of Congress and Courts, American Political Science Review 91(1):28-44.

Segal, Jeffrey A. and Albert D. Cover. 1989. Ideological Values and the Votes of U.S. Supreme Court Justices, American Political Science Review 83(2):557-565.

Segal, Jeffrey A. and Chad Westerland. 2005. The Supreme Court, Congress, and Judicial Review, North Carolina Law Review 83(5):1323-1352.

Segal, Jeffrey A., Chad Westerland and Stefanie A. Lindquist. 2011. Congress, the Supreme Court, and Judicial Review: Testing a Constitutional Separation of Powers Model, American Journal of Political Science 55(1):89-104.

Segal, Jeffrey A. and Harold J. Spaeth. 2002. The Supreme Court and the Attitudinal Model Revisited. New York: Cambridge University Press.

Segal, Jeffrey A., Lee Epstein, Charles M. Cameron, and Harold J. Spaeth. 1995. Ideological Values and the Votes of U.S. Supreme Court Justices Revisited, Journal of Politics 57(3):812-823.

Spiller, Pablo T. and Emerson H. Tiller. 1996. Invitations to Override: Congressional Reversals of Supreme Court Decisions, International Review of Law and Economics 16 (4): 503–521.

Stephenson, Matthew C. 2003. "When the Devil Turns . . . ": The Political Foundations of Independent Judicial Review, Journal of Legal Studies 32, 59-88.

Sunstein, Cass R. 2005. Radicals in Robes: Why Extreme Right-Wing Courts are Wrong for America, Cambridge MA: Basic Books.

Sunstein ,Cass R. 2009. A Constitution of Many Minds, Why the Founding Document Doesn't Mean What It Meant Before, Princeton: Princeton University Press.

Tushnet, Mark. 2005. A Court Divided: The Rehnquist Court and the Future of Constitutional Law. New York City, NY: WW Norton& Company.

Vanberg, Georg. 2015. Constitutional Courts in Comparative Perspective: A Theoretical Assessment, Annual Review of Political Science 18, 167-185.

Walhbeck, Paul J. 1997. The Life of the Law: Judicial Politics and Legal Change, Journal of Politics 59: 778-802.

APPENDIX WITH PROOFS

Proof Proposition 1: We are interested in comparing the pay-offs obtained by Blue when that coalition sets $s_1 = b$ or keeps $s_1 = s_0$ (coalitions are opposed). Those pay-offs are given by:

$$U_B(b) = \begin{cases} 1 - c + \delta\big(x + (1-x)(1-\Delta)\big) \text{ if } c < \Delta \\ 1 + \delta - c \text{ if } c \geq \Delta \end{cases}$$

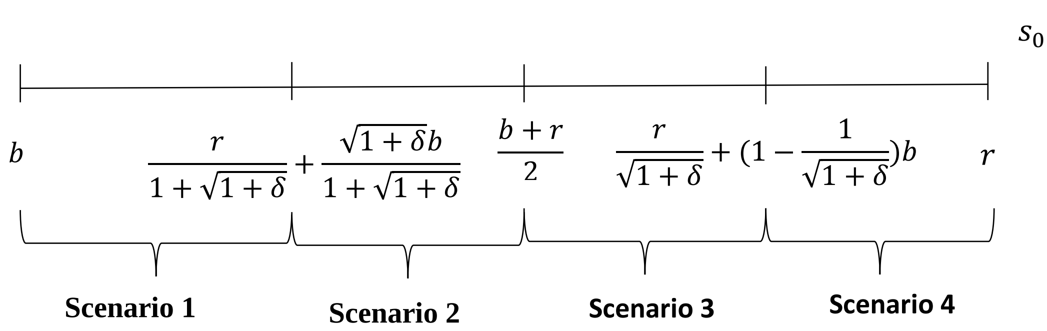
In addition, when $s_0 \in \left[b, \frac{r+b}{2}\right]$

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c < \Delta_b(s_0) \\ 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big) \text{ if } c \in [\Delta_b(s_0), \Delta_r(s_0)] \\ (1-\Delta_b(s_0))(1+\delta) \quad \text{if } c > \Delta_r(s_0) \end{cases}$$

But when $s_0 \in \left[\frac{r+b}{2}, r\right]$

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c < \Delta_r(s_0) \\ 1 - \Delta_b(s_0) + \delta\big((1-c)x + (1-x)(1-\Delta_b(s_0))\big) \text{ if } c \in [\Delta_r(s_0), \Delta_b(s_0)] \\ (1-\Delta_b(s_0))(1+\delta) \quad \text{if } c > \Delta_b(s_0) \end{cases}$$

To compare these expressions we discuss four scenarios that depend on the value of s_0.



Scenario 1: $s_0 \in \left[b, \frac{r}{1+\sqrt{1+\delta}} + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right)b\right] \rightarrow (s_0 - b)^2 < (1+\delta)(s_0 - b)^2 < (r - s_0)^2 < (r-b)^2$

If $c < (s_0 - b)^2$: b is a dominant strategy for Blue and r is a dominant strategy for Red

If $c \in [(s_0 - b)^2, (1+\delta)(s_0 - b)^2]$: Blue sets b only if x is large enough. Regardless whether Blue sets b or keeps s_0 then Red sets r. Then Blue sets b iff:

$$\left(1 - c + \delta(1 - (r-b)^2)\right)(1-x) + (1 - c + \delta)x$$
$$> \left((1 - (s_0 - b)^2) + \delta(1 - (r-b)^2)\right)(1-x)$$
$$+ (1 - (s_0 - b)^2)(1+\delta)x$$

$$\leftrightarrow -c + \delta x > -(s_0 - b)^2 + \delta x - (s_0 - b)^2 \delta x$$

$$\leftrightarrow c < (s_0 - b)^2 (1 + \delta x)$$

Which is true only if large enough. To see that, note that the inequality always hold when $x = 1$but it is never true when $x = 0$.

If $c \in [(1+\delta)(s_0 - b)^2, (r - s_0)^2]$: Regardless whether Blue sets b or keeps s_0 then Red sets r. Then Blue sets b iff:

$$\left(1 - c + \delta(1 - (r-b)^2)\right)(1-x) + (1 - c + \delta)x$$
$$> \left((1 - (s_0 - b)^2) + \delta(1 - (r-b)^2)\right)(1-x)$$
$$+ (1 - (s_0 - b)^2)(1+\delta)x$$

$$\leftrightarrow -c + \delta x > -(s_0 - b)^2 + \delta x - (s_0 - b)^2 \delta x$$

$$\leftrightarrow c < (s_0 - b)^2 (1 + \delta x)$$

Which is never true. Hence Blue keeps s_0 which is also kept by a Blue second period majority and a Red second period majority changes it to r.

If $c \in [(r - s_0)^2, (r - b)^2]$: Both coalitions keep the standard because the cost of a change is too high.

If $c > (r - b)^2$: Both coalitions keep the standard because the cost of a change is too high.

Scenario 2: $s_0 \in \left[\frac{r}{1+\sqrt{1+\delta}} + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b, \frac{r+b}{2}\right] \to (s_0 - b)^2 < (r - s_0)^2 < (1+\delta)(s_0 - b)^2 < (r - b)^2$

If $c < (s_0 - b)^2$: b is a dominant strategy for Blue and r is a dominant strategy for Red

If $c \in [(s_0 - b)^2, (r - s_0)^2]$: Blue sets b only if x is large enough. Indeed, regardless whether Blue sets b or keeps s_0 then Red sets r. Then Blue sets b iff:

$$\left(1 - c + \delta(1 - (r-b)^2)\right)(1-x) + (1 - c + \delta)x$$
$$> \left((1 - (s_0 - b)^2) + \delta(1 - (r-b)^2)\right)(1-x)$$
$$+ (1 - (s_0 - b)^2)(1+\delta)x$$

$$\leftrightarrow -c + \delta x > -(s_0 - b)^2 + \delta x - (s_0 - b)^2 \delta x$$

$$\leftrightarrow c < (s_0 - b)^2 (1 + \delta x)$$

Which is true only if x is large enough. To see that, note that the inequality always hold when $x = 1$ but it is never true when $x = 0$.

If $c \in [(r - s_0)^2, (1 + \delta)(s_0 - b)^2]$: Once more Blue sets b only if x is large enough. If Blue sets b then Red sets r and if B keeps s_0 then Red keeps that as well, then Blue sets b iff:

$$\begin{aligned}\big(1 - c + \delta(1 - (r - b)^2)\big)(1 - x) + (1 - c + \delta)x \\ > (1 - (s_0 - b)^2)(1 + \delta)(1 - x) + (1 - (s_0 - b)^2)(1 + \delta)x \\ = (1 - (s_0 - b)^2)(1 + \delta)\end{aligned}$$

$$\leftrightarrow -c - (r - b)^2(1 - x) > -(s_0 - b)^2(1 + \delta)$$

$$\leftrightarrow c < (s_0 - b)^2(1 + \delta) - \delta(r - b)^2(1 - x)$$

Which is true if and only if x is large enough. To see that note that the inequality always hold when $x = 1$ but it is never true when $x = 0$ because

$$(s_0 - b)^2(1 + \delta) - \delta(r - b)^2 < (s_0 - b)^2 \leftrightarrow (s_0 - b)^2 < (r - b)^2$$

If $c \in [(1 + \delta)(s_0 - b)^2, (r - b)^2]$: Both coalitions keep the standard because the cost of a change is too high.

If $c > (r - b)^2$: Both coalitions keep the standard because the cost of a change is too high.

Scenario 3: $s_0 \in \left[\frac{r+b}{2}, \frac{r}{\sqrt{1+\delta}} + \left(1 - \frac{1}{\sqrt{1+\delta}}\right)b\right] \rightarrow (r - s_0)^2 < (s_0 - b)^2 < (1 + \delta)(s_0 - b)^2 < (r - b)^2$

If $c < (r - s_0)^2$: b is dominant strategy for Blue and b is a dominant strategy for Red.

If $c \in [(r - s_0)^2, (s_0 - b)^2]$: Blue decides b at t = 1 only if x is large enough. To see that:

$$\begin{aligned}1 - c + \delta\big(x + (1 - x)(1 - (r - b)^2)\big) \\ > 1 - (s_0 - b)^2 + \delta((1 - c)x + (1 - x)(1 - (s_0 - b)^2))\end{aligned}$$

$$\leftrightarrow c < \frac{(s_0 - b)^2 - \delta[(r - b)^2 - (s_0 - b)^2](1 - x)}{1 - \delta x}$$

Which always hold when $x = 1$ because the inequality becomes $c < (s_0 - b)^2$ and only holds when c is small enough when $x = 0$. More specifically it only holds when

$$c < (s_0 - b)^2(1 + \delta) - \delta(r - b)^2$$

Note that we know that $(s_0 - b)^2(1 + \delta) - \delta(r - b)^2 < (s_0 - b)^2$ but $(s_0 - b)^2(1 + \delta) - \delta(r - b)^2 > (r - s_0)^2$ only when s_0 is large enough.

If $c \in [(s_0 - b)^2, (1 + \delta)(s_0 - b)^2]$: Once more Blue sets b only if x is large enough. If Blue sets b then Red sets r and if B keeps s_0 then Red keeps that as well, then Blue plays b iff:

$$\left(1 - c + \delta(1 - (r-b)^2)\right)(1-x) + (1 - c + \delta)x$$
$$> (1 - (s_0 - b)^2)(1+\delta)(1-x) + (1 - (s_0 - b)^2)(1+\delta)x$$
$$= (1 - (s_0 - b)^2)(1+\delta)$$
$$\leftrightarrow -c - (r-b)^2(1-x) > -(s_0 - b)^2(1+\delta)$$
$$\leftrightarrow c < (s_0 - b)^2(1+\delta) - \delta(r-b)^2(1-x)$$

Which is true if and only if x is large enough. To see that note that the inequality always hold when $x = 1$ but it is never true when $x = 0$ because

$$(s_0 - b)^2(1+\delta) - (r-b)^2 < (s_0 - b)^2 \leftrightarrow (s_0 - b)^2 < (r-b)^2$$

If $c \in [(1+\delta)(s_0 - b)^2, (r-b)^2]$: Both coalitions keep the standard because the cost of a change is too high. To see that

$$1 - c + \delta\left(x + (1-x)(1 - (r-b)^2)\right) < (1 - (s_0 - b)^2)(1+\delta)$$
$$\leftrightarrow c > (s_0 - b)^2(1+\delta) - \delta(1-x)(r-b)^2$$

Which is always true when $c \in [(1+\delta)(s_0 - b)^2, (r-b)^2]$.

If $c > (r-b)^2$: Both coalitions keep the standard because the cost of a change is too high.

Scenario 4: $s_0 \in \left[\frac{r}{\sqrt{1+\delta}} + \left(1 - \frac{1}{\sqrt{1+\delta}}\right)b, r\right] \to (r - s_0)^2 < (s_0 - b)^2 < (r-b)^2 < (1+\delta)(s_0 - b)^2$

If $c < (r - s_0)^2$: Knowing that Red always decide r at t = 2 then Blue always set b. Formally, it is direct that

$$1 - c + \delta\left(x + (1-x)(1 - (r-b)^2)\right)$$
$$> 1 - (s_0 - b)^2 + \delta((1-c)x + (1-x)(1 - (r-b)^2))$$

Because $c < (s_0 - b)^2$ and $1 - c < 1$.

If $c \in [(r - s_0)^2, (s_0 - b)^2]$: Blue decides b at t = 1 only if x is large enough. To see that:

$$1 - c + \delta\left(x + (1-x)(1 - (r-b)^2)\right)$$
$$> 1 - (s_0 - b)^2 + \delta((1-c)x + (1-x)(1 - (s_0 - b)^2))$$
$$\leftrightarrow c < \frac{(s_0 - b)^2 - \delta[(r-b)^2 - (s_0 - b)^2](1-x)}{1 - \delta x}$$

Which always hold when $x = 1$ because the inequality becomes $c < (s_0 - b)^2$ and only holds when c is small enough when $x = 0$. More specifically it only holds when

$$c < (s_0 - b)^2(1+\delta) - \delta(r-b)^2$$

Note that we know that $(s_0 - b)^2(1+\delta) - \delta(r-b)^2 < (s_0 - b)^2$ but $(s_0 - b)^2(1+\delta) - \delta(r-b)^2 > (r - s_0)^2$ only when s_0 is large enough.

If $c \in [(s_0 - b)^2, (r-b)^2]$: Once more Blue sets b only if x is large enough. If Blue sets b at t = 1 then Red sets r at t = 2 and if B keeps s_0 at t = 1 then Red also keeps that, then Blue sets b iff:

$$1 - c + \delta\big(x + (1-x)(1-(r-b)^2)\big) > (1-(s_0-b)^2)(1+\delta)$$

$$\leftrightarrow -c - \delta(r-b)^2(1-x) > -(s_0-b)^2(1+\delta)$$

$$\leftrightarrow c < (s_0-b)^2(1+\delta) - \delta(r-b)^2(1-x)$$

Which is true if and only if x is large enough. To see that note that the inequality always hold when $x = 1$ but it is never true when $x = 0$ because

$$(s_0-b)^2(1+\delta) - \delta(r-b)^2 < (s_0-b)^2 \leftrightarrow (s_0-b)^2 < (r-b)^2$$

If $c \in [(r-b)^2, (1+\delta)(s_0-b)^2]$: Red always keep the standard and because of that Blue sets b at t = 1. Because that is also dominant when Blue is majority in the second period then Blue sets b and Red keeps b. To see that:

$$1 + \delta - c > (1-(s_0-b)^2)(1+\delta)$$

$$\leftrightarrow c < (s_0-b)^2(1+\delta)$$

If $c > (1+\delta)(s_0-b)^2$: Both coalitions always keep s_0 because the cost is too high.

Summarizing:

If $s_0 > \frac{r+b}{2}$ then $\underline{c} = (1+\delta)\Delta_b(s_0) - \delta\Delta$ and $\overline{c} = (1+\delta)\Delta_b(s_0)$.

If $s_0 < \frac{r+b}{2}$ then $\underline{c} = \Delta_b(s_0)$ and $\overline{c} = (1+\delta)\Delta_b(s_0)$.

In addition, after we separate by scenario, $x^*(c)$ is equal to

In scenario 1:

$$x^*(c) = \frac{c - \delta\Delta_b(s_0)}{\delta\Delta_b(s_0)} \text{ when } c \in [\Delta_b(s_0), (1+\delta)\Delta_b(s_0)]$$

In scenario 2:

$$x^*(c) = \begin{cases} \dfrac{c - \delta\Delta_b(s_0)}{\delta\Delta_b(s_0)} \text{ when } c \in [\Delta_b(s_0), \Delta_r(s_0)] \\ \dfrac{c - \big((1+\delta)\Delta_b(s_0) - \delta\Delta\big)}{\delta\Delta} \text{ when } c \in [\Delta_r(s_0), (1+\delta)\Delta_b(s_0)] \end{cases}$$

In scenario 3:

$$x^*(c) = \begin{cases} \dfrac{c - \big((1+\delta)\Delta_b(s_0) - \delta\Delta\big)}{\delta(\Delta - \Delta_b(s_0) + c)} \text{ when } c \in [(1+\delta)\Delta_b(s_0) - \delta\Delta, \Delta_b(s_0)] \\ \dfrac{c - \big((1+\delta)\Delta_b(s_0) - \delta\Delta\big)}{\delta\Delta} \text{ when } c \in [\Delta_b(s_0), (1+\delta)\Delta_b(s_0)] \end{cases}$$

In scenario 4:

$$x^*(c) = \begin{cases} \dfrac{c - \big((1+\delta)\Delta_b(s_0) - \delta\Delta\big)}{\delta(\Delta - \Delta_b(s_0) + c)} \text{ when } c \in [(1+\delta)\Delta_b(s_0) - \delta\Delta\,, \Delta_b(s_0)] \\ \dfrac{c - \big((1+\delta)\Delta_b(s_0) - \delta\Delta\big)}{\delta\Delta} \text{ when } c \in [\Delta_b(s_0), \Delta] \\ 0 \text{ when } c \in [\Delta, (1+\delta)\Delta_b(s_0)] \end{cases}$$

In all these cases $\frac{\partial x^*(c)}{\partial c} \geq 0$. That proves the desired results. **End of the Proof.**

Proof of Proposition 2: We are interested in comparing the pay-offs obtained by Blue when that coalition sets $s_1 = b$ or ke eps $s_1 = s_0$ (coalitions are aligned). Those pay-offs are given by:

$$U_B(b) = \begin{cases} 1 - c + \delta\big(x + (1-x)(1-\Delta)\big) \text{ if } c < \Delta \\ 1 + \delta - c \text{ if } c \geq \Delta \end{cases}$$

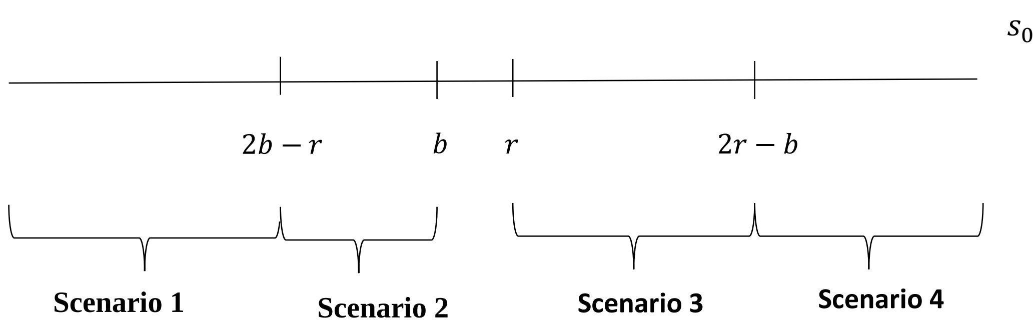
In addition, when $s_0 < b < r$ then

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c < \Delta_b(s_0) \\ 1 - \Delta_b(s_0) + \delta\big(x(1 - \Delta_b(s_0)) + (1-x)(1-\Delta)\big) \text{ if } c \in [\Delta_b(s_0), \Delta_r(s_0)] \\ (1 - \Delta_b(s_0))(1+\delta) \quad \text{if } c > \Delta_r(s_0) \end{cases}$$

But when $b < r < s_0$ then

$$U_B(s_0) = \begin{cases} 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta)) \text{ if } c < \Delta_r(s_0) \\ 1 - \Delta_b(s_0) + \delta\big((1-c)x + (1-x)(1 - \Delta_b(s_0))\big) \text{ if } c \in [\Delta_r(s_0), \Delta_b(s_0)] \\ (1 - \Delta_b(s_0))(1+\delta) \quad \text{if } c > \Delta_b(s_0) \end{cases}$$

To compare these expressions we discuss four scenarios that depend on the value of s_0.



Scenario 1: $s_0 < 2b - r < b < r \to \Delta < \Delta_b(s_0) < \Delta_r(s_0)$

If $c < \Delta$: b is a dominant strategy for Blue and r is a dominant strategy for Red which implies that Blue sets b at t = 1. To see that

$$1 - c + \delta\big(x + (1-x)(1-\Delta)\big) > 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta))$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)}{1-\delta x}$$

Which evidently holds.

If $c \in [\Delta, \Delta_b(s_0)]$: Once more Blue sets b for all values of x. Indeed Blue sets b iff:

$$1 + \delta - c > 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big)$$

Which is always true because $x(1-\Delta_b(s_0)) + (1-x)(1-\Delta) < 1$ and $c < \Delta_b(s_0)$.

If $c \in [\Delta_b(s_0), \Delta_r(s_0)]$: Blue sets b only if x is large enough otherwise it keeps s_0. Indeed Blue sets b iff:

$$1 + \delta - c > 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big)$$

$$\leftrightarrow c < (1+\delta x)\Delta_b(s_0) + \delta(1-x)\Delta$$

$$\leftrightarrow x > \frac{c - (\Delta_b(s_0) + \delta\Delta)}{\delta(\Delta_b(s_0) - \Delta)}$$

Which holds for all x when $c = \Delta_b(s_0)$ but it never holds for $c \in [(1+\delta)\Delta_b(s_0), \Delta_r(s_0)]$ because the RHS becomes 1 when $c = (1+\delta)\Delta_b(s_0)$.

If $c > \Delta_r(s_0)$: Both coalitions keep the standard because the cost of a change is too high. To see that

$$1 + \delta - c < \big(1 - \Delta_b(s_0)\big)(1+\delta) \leftrightarrow c > (1+\delta)\Delta_b(s_0)$$

Note that $(1+\delta)\Delta_b(s_0) < \Delta_r(s_0) \leftrightarrow s_0 \in \left[\frac{\sqrt{1+\delta}}{\sqrt{1+\delta}-1}b - \frac{r}{\sqrt{1+\delta}-1}, 2b - r\right]$.

Scenario 2: $2b - r < s_0 < b < r \to \Delta_b(s_0) < \Delta < \Delta_r(s_0)$

If $c < \Delta_b(s_0)$: b is a dominant strategy for Blue and r is a dominant strategy for Red which implies that Blue sets b at t = 1. To see that

$$1 - c + \delta\big(x + (1-x)(1-\Delta)\big) > 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta))$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)}{1-\delta x}$$

Which evidently always hold.

If $c \in [\Delta_b(s_0), \Delta]$: Blue sets b only when x is large enough. To see that, Blue sets b iff:

$$1 - c + \delta\big(x + (1-x)(1-\Delta)\big) > 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big)$$

$$\leftrightarrow c < (1+\delta x)\Delta_b(s_0) \leftrightarrow x > \frac{c - \Delta_b(s_0)}{\delta \Delta_b(s_0)}$$

Note that if $s_0 > \left(\frac{1+\sqrt{1+\delta}}{\sqrt{1+\delta}}\right) b - \frac{1}{\sqrt{1+\delta}} r \in [2b-r, b]$ then $(1+\delta)\Delta_b(s_0) < \Delta$ and then for $c \in [(1+\delta)\Delta_b(s_0), \Delta]$ Blue prefers b to s_0 for all values of x. But if $s_0 < \left(\frac{1+\sqrt{1+\delta}}{\sqrt{1+\delta}}\right) b - \frac{1}{\sqrt{1+\delta}} r$ then $(1+\delta)\Delta_b(s_0) > \Delta$ and Blue prefers b iff x is large enough.

If $c \in [\Delta, \Delta_r(s_0)]$: Blue sets b only if x is small enough otherwise it keeps s_0. Indeed Blue sets b iff:

$$1 + \delta - c > 1 - \Delta_b(s_0) + \delta\big(x(1-\Delta_b(s_0)) + (1-x)(1-\Delta)\big)$$

$$\leftrightarrow c < (1+\delta x)\Delta_b(s_0) + \delta(1-x)\Delta$$

$$\leftrightarrow x < \frac{(\Delta_b(s_0) + \delta\Delta) - c}{\delta(\Delta - \Delta_b(s_0))}$$

Which holds for all x when $c = (1+\delta)\Delta_b(s_0)$ because the RHS becomes 1 but it never holds for $c > \Delta_b(s_0) + \delta\Delta$ because the RHS becomes 0. Note that $(1+\delta)\Delta_b(s_0) < \Delta_r(s_0)$ because that is equivalent to $s_0 > \frac{\sqrt{1+\delta}}{\sqrt{1+\delta}-1} b - \frac{1}{\sqrt{1+\delta}-1} r$ which is true because

$$\frac{\sqrt{1+\delta}}{\sqrt{1+\delta}-1} b - \frac{1}{\sqrt{1+\delta}-1} r < 2b - r \leftrightarrow r > b$$

If $c > \Delta_r(s_0)$: Both coalitions keep the standard because the cost of a change is too high. To see that

$$1 + \delta - c < \big(1 - \Delta_b(s_0)\big)(1+\delta) \leftrightarrow c > (1+\delta)\Delta_b(s_0)$$

Which is always true given that $(1+\delta)\Delta_b(s_0) < \Delta_r(s_0)$.

Scenario 3: $b < r < s_0 < 2r - b \rightarrow \Delta_r(s_0) < \Delta < \Delta_b(s_0)$

If $c < \Delta_r(s_0)$: b is a dominant strategy for Blue and r is a dominant strategy for Red which implies that Blue sets b at t = 1. To see that

$$1 - c + \delta\big(x + (1-x)(1-\Delta)\big) > 1 - \Delta_b(s_0) + \delta((1-c)x + (1-x)(1-\Delta))$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)}{1 - \delta x}$$

Which evidently always hold.

If $c \in [\Delta_r(s_0), \Delta]$: Blue always set b. To see that, Blue sets b iff:

$$1-c+\delta\big(x+(1-x)(1-\Delta)\big) > 1-\Delta_b(s_0)+\delta\big((1-c)x+(1-x)(1-\Delta_b(s_0))\big)$$

$$\leftrightarrow -c-\delta(1-x)\Delta > -\Delta_b(s_0)-\delta cx-\delta(1-x)\Delta_b(s_0)$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)+(1-x)\delta(\Delta_b(s_0)-\Delta)}{1-\delta x}$$

Which is always true because we are in a range in which $c < \Delta_b(s_0)$.

If $c \in [\Delta, \Delta_b(s_0)]$: Blue sets b for all values of x because:

$$1+\delta-c > 1-\Delta_b(s_0)+\delta\left((1-c)x+(1-x)\big(1-\Delta_b(s_0)\big)\right)$$

$$\leftrightarrow -c > -\Delta_b(s_0)-\delta cx-\delta(1-x)\Delta_b(s_0)$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)+(1-x)\delta\Delta_b(s_0)}{1-\delta x}$$

Which is always true because we are in a range in which $c < \Delta_b(s_0)$.

If $c > \Delta_b(s_0)$: Both coalitions keep the standard if the cost is large enough otherwise they set b. To see that, Blue sets b iff

$$1+\delta-c < \big(1-\Delta_b(s_0)\big)(1+\delta) \leftrightarrow c > (1+\delta)\Delta_b(s_0)$$

Scenario 4: $b < r < 2r-b < s_0 \rightarrow \Delta < \Delta_r(s_0) < \Delta_b(s_0)$

If $c < \Delta$: b is a dominant strategy for Blue and r is a dominant strategy for Red which implies that Blue sets b at t = 1. To see that

$$1-c+\delta\big(x+(1-x)(1-\Delta)\big) > 1-\Delta_b(s_0)+\delta((1-c)x+(1-x)(1-\Delta))$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)}{1-\delta x}$$

Which evidently always hold.

If $c \in [\Delta, \Delta_r(s_0)]$: Blue always sets b. To see that, Blue sets b iff:

$$1-c+\delta\big(x+(1-x)(1-\Delta)\big) > 1-\Delta_b(s_0)+\delta\big((1-c)x+(1-x)(1-\Delta_b(s_0))\big)$$

$$\leftrightarrow -c-\delta(1-x)\Delta > -\Delta_b(s_0)-\delta cx-\delta(1-x)\Delta_b(s_0)$$

$$\leftrightarrow c < \frac{\Delta_b(s_0)+\delta(1-x)\Delta}{1-\delta x}$$

Which is always true because we are in a range in which $c < \Delta_b(s_0)$.

If $c \in [\Delta_r(s_0), \Delta_b(s_0)]$: Blue sets b for all values of x because:

$$1+\delta-c > 1-\Delta_b(s_0)+\delta\left((1-c)x+(1-x)\big(1-\Delta_b(s_0)\big)\right)$$

$$\leftrightarrow -c > -\Delta_b(s_0) - \delta c x - \delta(1-x)\Delta_b(s_0)$$

$$\leftrightarrow c < \frac{\Delta_b(s_0) + (1-x)\delta\Delta_b(s_0)}{1-\delta x}$$

Which is always true because we are in a range in which $c < \Delta_b(s_0)$.

If $c > \Delta_b(s_0)$: Both coalitions keep the standard if the cost is large enough otherwise they set b. To see that, Blue sets b iff

$$1 + \delta - c < \left(1 - \Delta_b(s_0)\right)(1+\delta) \leftrightarrow c > (1+\delta)\Delta_b(s_0)$$

Summarizing:

If $s_0 < 2b - r$ then $\underline{c} = \Delta_b(s_0)$ and $\overline{c} = \Delta_r(s_0)$.

If $s_0 \in [2b - r, b]$ then $\underline{c} = \Delta_b(s_0)$ and $\overline{c} = \Delta$ and $\overline{\overline{c}} = \Delta_r(s_0)$

If $s_0 > r$ then $\underline{c} = \overline{c} = (1+\delta)\Delta_b(s_0)$.

In addition, after we separate by scenario, $x^*(c)$ is equal to

In scenario 1:

$$x^*(c) = \frac{c - (\Delta_b(s_0) + \delta\Delta)}{\delta(\Delta_b(s_0) - \Delta)} \text{ when } c \in [\Delta_b(s_0), \Delta_r(s_0)]$$

In scenario 2:

$$x^*(c) = \frac{c - \Delta_b(s_0)}{\delta\Delta_b(s_0)} \text{ when } c \in [\Delta_b(s_0), \Delta]$$

And

$$x_1^*(c) = \frac{(\Delta_b(s_0) + \delta\Delta) - c}{\delta\left(\Delta - \Delta_b(s_0)\right)} \text{ when } c \in [\Delta, \Delta_r(s_0)]$$

It is direct that $\frac{\partial x^*(c)}{\partial c} > 0$ and $\frac{x_1^*(c)}{\partial c} < 0$. That proves the desired results. **End of the Proof.**

Proof of Corollary 1: In order to retrieve all the cases we need to identify the values for c_1, c_2, c_3, c_4 and also s_1, s_2 associated to the four scenarios discussed in Proposition 1.

When $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta}} r + (1 - \frac{1}{1+\sqrt{1+\delta}}) b\right]$ then:

Cost	$c_1 = \Delta_b(s_0)$		$c_2 = c_3 = c_4 = \Delta_r(s_0)$
t = 1	Blue sets b	Blue sets b iff x is large enough	Blue keeps s_0
t = 2	Blue keeps s_1 Red sets r	Blue keeps s_1 Red sets r if $s_1 = b$ but keeps $s_1 = s_0$	Blue keeps s_0

When $s_0 \in \left[\frac{1}{1+\sqrt{1+\delta}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b, \frac{r+b}{2}\right]$ then:

Cost	$c_1 = \Delta_b(s_0)$	$c_2 = \Delta_r(s_0)$	$c_3 = c_4 = (1+\delta)\Delta_b(s_0)$	
t = 1	Blue sets b	Blue sets b iff x is large enough	Blue sets b iff x is large enough	Blue keeps s_0
t = 2	Blue keeps s_1 Red sets r	Blue keeps s_1 Red sets r	Blue keeps s_1 Red sets r if $s_1 = b$ but keeps $s_1 = s_0$	Blue keeps s_0 Red keeps s_0

When $s_0 \in \left[\frac{b+r}{2}, \frac{1}{\sqrt{1+\delta}} r + \left(1 - \frac{1}{\sqrt{1+\delta}}\right) b\right]$ then

Cost

$c_1 = c_2 = (1+\delta)\Delta_b(s_0) - \delta\Delta$ $\quad$ $c_3 = c_4 = (1+\delta)\Delta_b(s_0)$

t = 1	Blue sets b	Blue sets b iff x is large enough	Blue keeps s_0
t = 2	Blue keeps s_1 Red sets r	Blue keeps s_1 Red sets r if $s_1 = b$ but keeps $s_1 = s_0$	Blue keeps s_0 Red keeps s_0

Finally when $s_0 \in \left[\frac{1}{\sqrt{1+\delta}} r + \left(1 - \frac{1}{\sqrt{1+\delta}}\right) b, r\right]$ then

Cost

$c_1 = c_2 = (1+\delta)\Delta_b(s_0) - \delta\Delta$ $\quad$ $c_3 = \Delta$ $\quad$ $c_4 = (1+\delta)\Delta_b(s_0)$

t = 1	Blue sets b	Blue sets b iff x is large enough	Blue sets b	Blue keeps s_0
t = 2	Blue keeps s_1 Red sets r	Blue keeps s_1 Red sets r if $s_1 = b$ but keeps $s_1 = s_0$	Blue keeps s_1 Red keeps s_1	Blue keeps s_0 Red keeps s_0

End of Proof.

Proof of Corollary 2: We discuss the same four scenarios presented in proposition 1 separately. We base our results on the characterization of the strategies followed by the coalitions in the two periods presented in Corollary 1.

Scenario 1: From inspection of Corollary 1 the only case in which both coalitions always keep s_0 is when $c > \Delta_r(s_0)$.

Scenario 2: From inspection of Corollary 1 both coalitions always keep s_0 in two cases. First, when $c > (1+\delta)\Delta_b(s_0)$ and second, when $c \in [\Delta_r(s_0), (1+\delta)\Delta_b(s_0)]$ in addition to $x < \frac{c-((1+\delta)\Delta_b(s_0)-\delta\Delta)}{\delta\Delta}$.

Scenario 3: From inspection of Corollary 1 both coalitions always keep s_0 in two cases. First, when $c > (1+\delta)\Delta_b(s_0)$ and second, when $c \in [(1+\delta)\Delta_b(s_0) - \delta\Delta, (1+\delta)\Delta_b(s_0)]$ in addition to $x < \frac{c-((1+\delta)\Delta_b(s_0)-\delta\Delta)}{\delta\Delta}$.

Scenario 4: From inspection of Corollary 1 both coalitions always keep s_0 in two cases. First, when $c > (1+\delta)\Delta_b(s_0)$ and second, when $c \in [\Delta_b(s_0), \Delta]$ in addition to $x < \frac{c-((1+\delta)\Delta_b(s_0)-\delta\Delta)}{\delta\Delta}$.

That retrieves the of results presented in Corollary 2 with $\underline{s_0} = \frac{1}{1+\sqrt{1+\delta}} r + (1 - \frac{1}{1+\sqrt{1+\delta}}) b$ and $\overline{s_0} = \frac{1}{\sqrt{1+\delta}} r + \left(1 - \frac{1}{\sqrt{1+\delta}}\right) b$. In addition $\overline{c}$ might take the value of $\Delta_r(s_0)$, $(1+\delta)\Delta_b(s_0) - \delta\Delta$ or $\Delta_b(s_0)$, $\overline{\overline{c}} = \Delta$, $\overline{\overline{\overline{c}}} = (1+\delta)\Delta_b(s_0)$. Finally $x^*(c) = \frac{c-((1+\delta)\Delta_b(s_0)-\delta\Delta)}{\delta\Delta}$. **End of Proof.**

Proof of Proposition 3: Suppose that Blue is majority at $t = 1$ and remains as majority at any period $t \in \{2,3\}$ with probability x.[25] In order to keep the analysis as simple as possible,

[25] Results do not change if we consider that this probability changes every period but the analysis becomes considerably more cumbersome.

we impose that once a coalition changes the standard from s_0 it never goes back to that value.[26] As we did before, here we only discuss the case in which s_0 is very close to b, that is $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta}} r + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$ which implies $\Delta_b(s_0) < \Delta_r(s_0)$. The main messages do not change if we allow the initial standard to get closer to r.[27] The first point to notice is that *maximum stability* cannot take place when $c < \Delta_r(s_0)$ because a third period Red majority changes s_0 to r. In addition, we know that maximum stability always take place when $c > \Delta$ because the cost of making any change is too expensive for any coalition. Next we discuss what happens when $c \in [\Delta_r(s_0), \Delta]$.

It is direct that at t = 3 both coalitions set their preferred standards when they face the opposite standard (because $c < \Delta$) and both coalitions keep the standard when they face s_0 (because $c > \Delta_r(s_0) > \Delta_b(s_0)$). The analysis is more involved at t = 2, then the coalitions face two options: $s_1 = b$ or $s_1 = s_0$. If $s_1 = b$ then Blue keeps the standard but Red changes it to r. Indeed, when Red sets r it gets

$$1 - c + \delta(x(1 - \Delta) + (1 - x))$$

While if it keeps b it gets

$$1 - \Delta + \delta(x(1 - \Delta) + (1 - x)(1 - c))$$

And the first expression is larger than the second because $c < \frac{\Delta}{(1-\delta x)}$ holds for all values of x.[28] If $s_1 = s_0$ then Blue keeps the standard only when x is small enough and Red does it only when x is large enough. To see that, if Blue sets b it gets

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[26] The discussion of the 3-period model is not that different without this assumption but the complexity of the T-period model increases considerably.

[27] That said, the cleanest effects can be identified when s_0 takes intermediate values (s_0 gets closer to $\frac{r+b}{2}$).

[28] Note that Red also prefers r to s_0 because it gets $1 - \Delta_r(s_0) - c + \delta\left(x\left(1 - \Delta_r(s_0)\right) + (1 - x)\left(1 - \Delta_r(s_0)\right)\right)$ when it sets s_0 which is evidently smaller than $1 - c + \delta(x(1 - \Delta) + (1 - x))$.

$$1-c+\delta(x+(1-x)(1-\Delta))$$

While if it keeps s_0 it gets

$$1-\Delta_b(s_0)+\delta(x(1-\Delta_b(s_0))+(1-x)(1-\Delta_b(s_0)))$$

The first expression is larger only when

$$c<(1+\delta)\Delta_b(s_0)-\delta(1-x)\Delta \qquad (5)$$

which never holds when s_0 is very close to b (more specifically when $s_0 \in \left[b, \frac{1}{1+\sqrt{1+\delta(1+\delta)}}r+\left(1-\frac{1}{1+\sqrt{1+\delta(1+\delta)}}\right)b\right]$) but might hold when $s_0 \in \left[\frac{1}{1+\sqrt{1+\delta(1+\delta)}}r+\left(1-\frac{1}{1+\sqrt{1+\delta(1+\delta)}}\right)b, \frac{1}{1+\sqrt{1+\delta}}r+\left(1-\frac{1}{1+\sqrt{1+\delta}}\right)b\right]$. In the case of Red, if it sets r it gets

$$1-c+\delta(x(1-\Delta)+(1-x))$$

While if it keeps s_0 it gets

$$1-\Delta_r(s_0)+\delta(x(1-\Delta_r(s_0))+(1-x)(1-\Delta_r(s_0)))$$

The first expression is greater only when

$$c<(1+\delta)\Delta_r(s_0)-\delta x\Delta \qquad (6)$$

which means that Red sets r only when x is small enough.[29] Finally at t = 1 Blue has to decide whether to set $s_1=b$ or keep $s_1=s_0$ knowing the strategies implemented at t = 2. If

[29] Note that the RHS of (5) is decreasing in x.

(5) holds and (6) does not hold then Blue and Red keep s_0 at t = 2 and Blue sets b instead of s_0 if and only if

$$1 - c + \delta x\big(1 + \delta(x + (1-x)(1-\Delta)\big)$$
$$+ \delta(1-x)\Big(1 - \Delta + \delta\big(x(1-c) + (1-x)(1-\Delta)\big)\Big)$$
$$> \big(1 - \Delta_b(s_0)\big)(1 + \delta(1+\delta))$$

Hence Blue sets $s_1 = b$ if and only if

$$c < \frac{\Delta_b(s_0)\big(1 + \delta(1+\delta)\big) - \delta\Delta(1-x))}{(1 + \delta^2 x(1-x))} \qquad (7)$$

Because (7) is more demanding than (5), if (6) and (7) do not hold then both Blue and Red keep the original standard in the second period. In other words, there is maximum stability when

$$x > \underline{x}(c) = \frac{(1+\delta)\Delta_r(s_0) - c}{\delta\Delta}$$

and when

$$x < \overline{x}(c) = \frac{\delta c - \Delta + \sqrt{(\delta c - \Delta)^2 - 4\big(\Delta_b(s_0)\big(1+\delta(1+\delta)\big) - \delta\Delta - c\big)c}}{2\delta c}.$$

Evidently, *maximum stability* cannot take place when (5) holds. **End of Proof.**

FIGURES

Figure 1.1.O Majority decision at t = 1 when $s_0 \in \left[b, \frac{r}{1+\sqrt{1+\delta}} + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$.

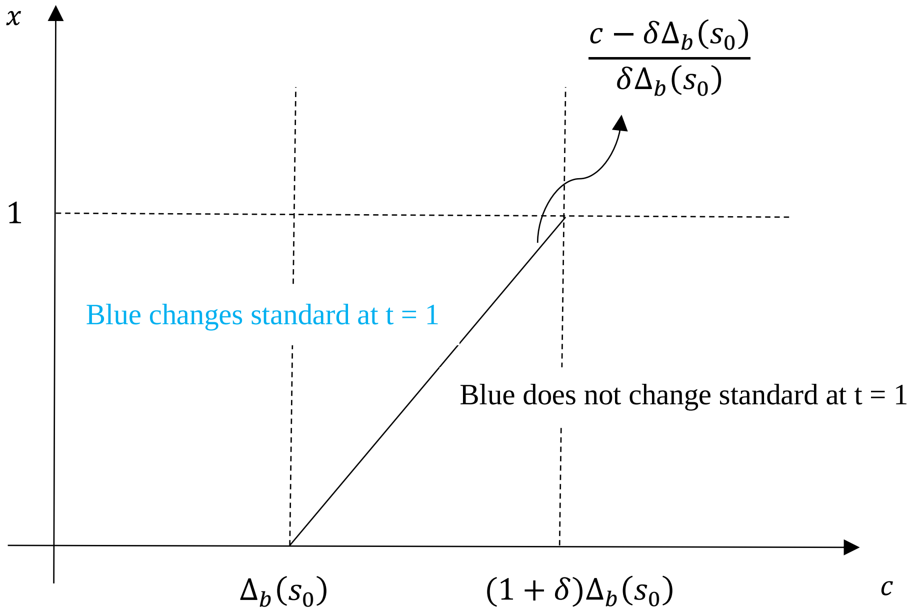


Figure 1.2.O Majority decision at t = 1 when $s_0 \in \left[\frac{r}{1+\sqrt{1+\delta}} + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b, \frac{r+b}{2}\right]$

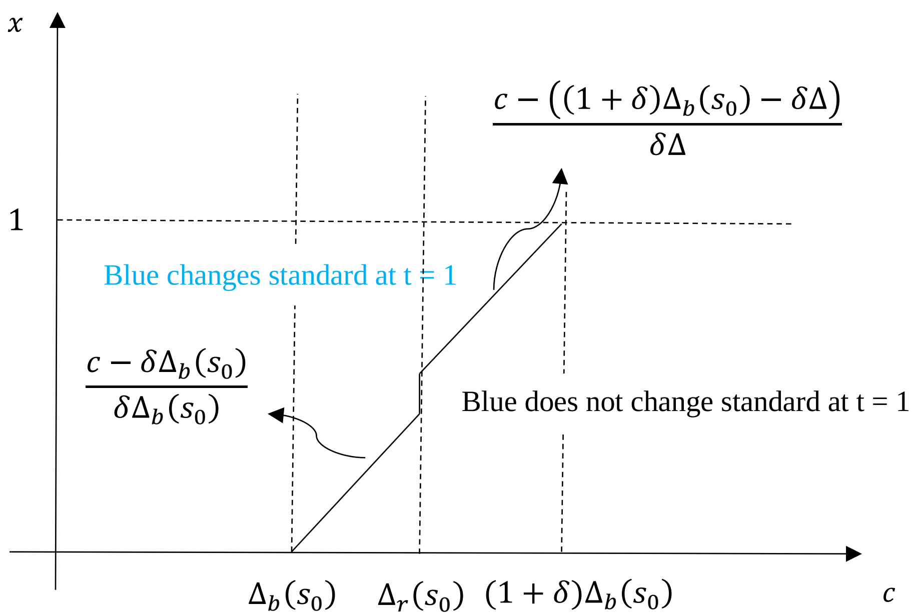


Figure 1.3.O Majority decision at t = 1 when $s_0 \in \left[\frac{r+b}{2}, \frac{r}{\sqrt{1+\delta}} + \left(1 - \frac{1}{\sqrt{1+\delta}}\right) b\right]$

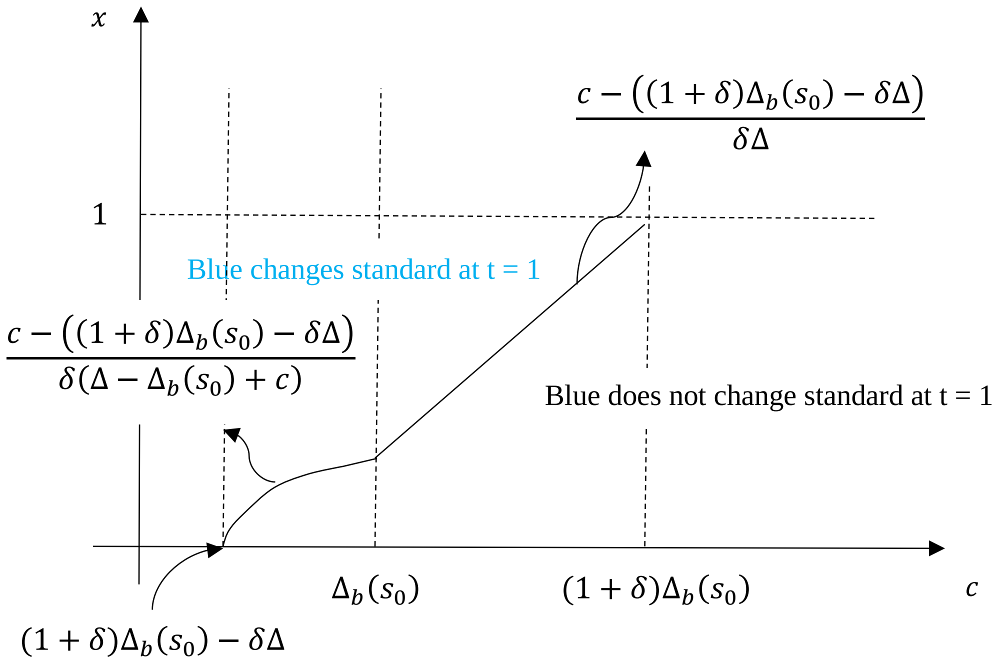


Figure 1.4.O Majority decision at t = 1 when $s_0 \in \left[\frac{r}{\sqrt{1+\delta}} + \left(1 - \frac{1}{\sqrt{1+\delta}}\right) b, r\right]$

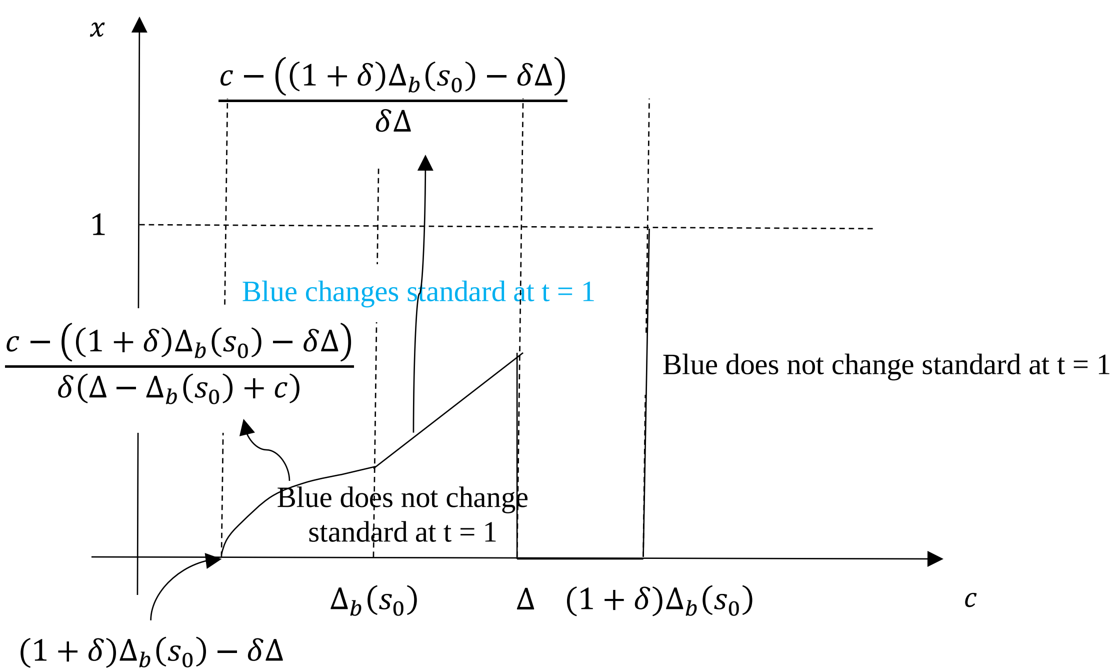


Figure 1.1.A.I Majority decision at t = 1 when $s_0 < \frac{\sqrt{1+\delta}}{\sqrt{1+\delta}-1}b - \frac{r}{\sqrt{1+\delta}-1}$.

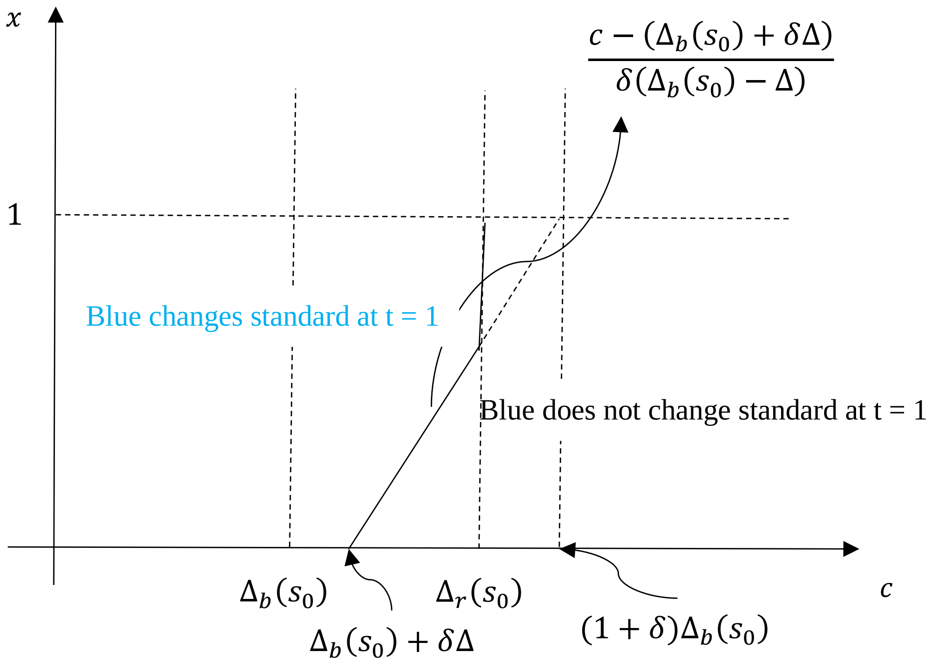


Figure 1.1.A.II Majority decision at t = 1 when $s_0 \in \left[\frac{\sqrt{1+\delta}}{\sqrt{1+\delta}-1}b - \frac{r}{\sqrt{1+\delta}-1}, 2b - r\right]$

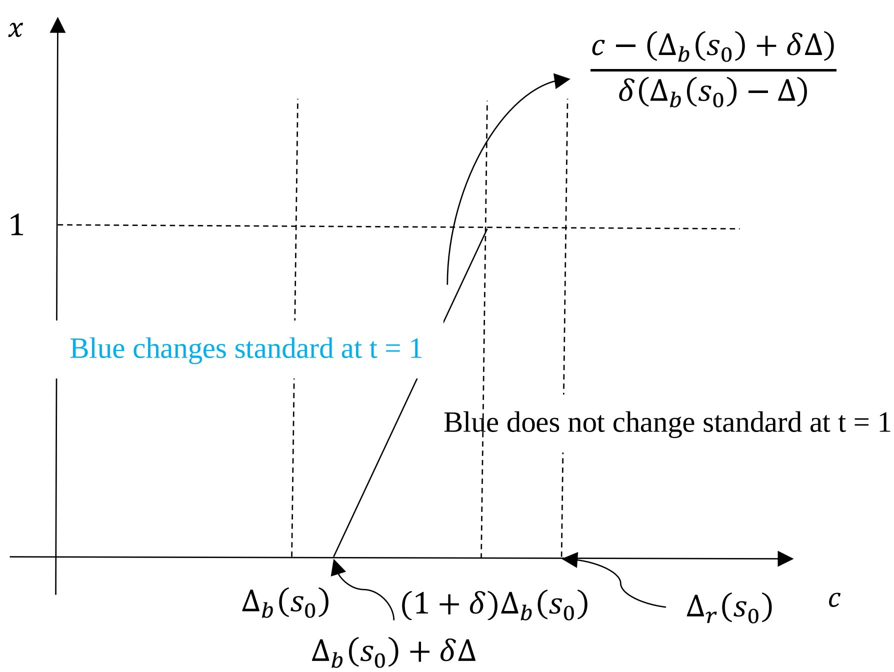


Figure 1.2.A.I Majority decision at t = 1 when $s_0 \in \left[2b - r, \left(\frac{1+\sqrt{1+\delta}}{\sqrt{1+\delta}}\right)b - \frac{1}{\sqrt{1+\delta}}r\right]$

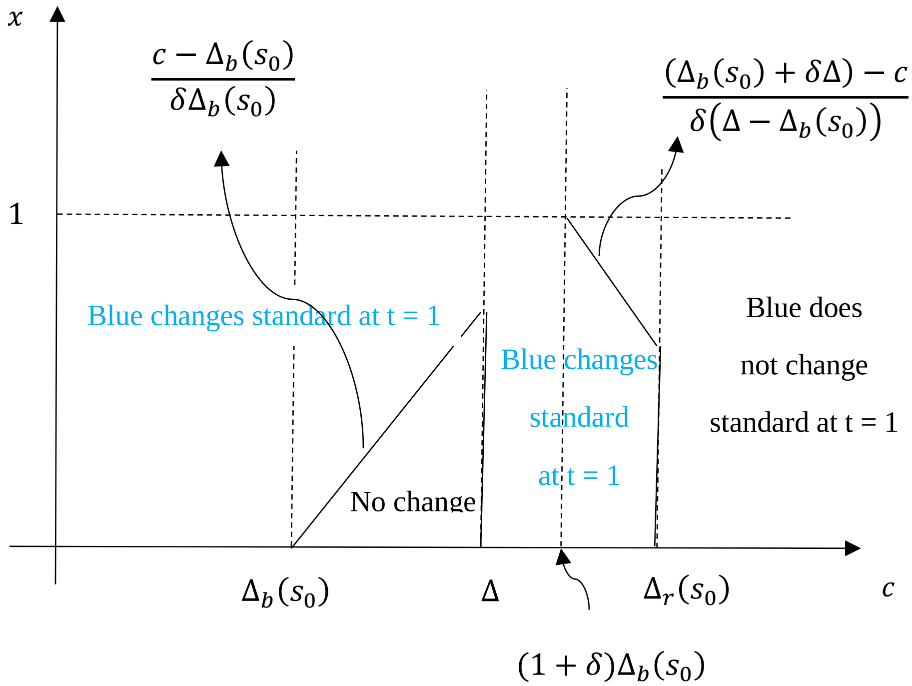


Figure 1.2.A.II Majority decision at t = 1 when $s_0 \in \left[\left(\frac{1+\sqrt{1+\delta}}{\sqrt{1+\delta}}\right)b - \frac{1}{\sqrt{1+\delta}}r, b\right]$

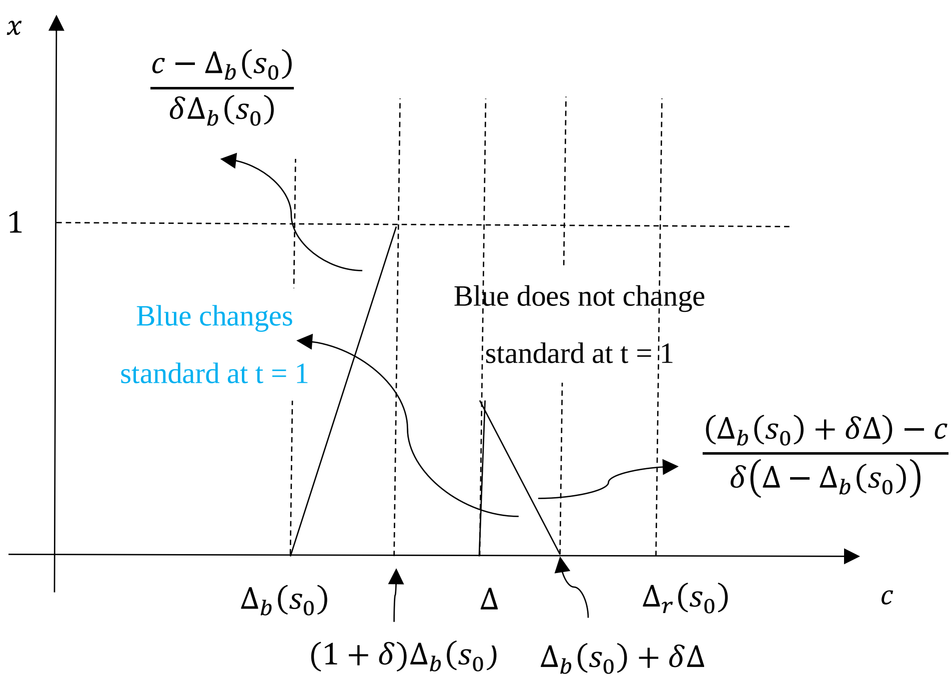


Figure 1.3.A Majority decision at t = 1 when $s_0 > r$

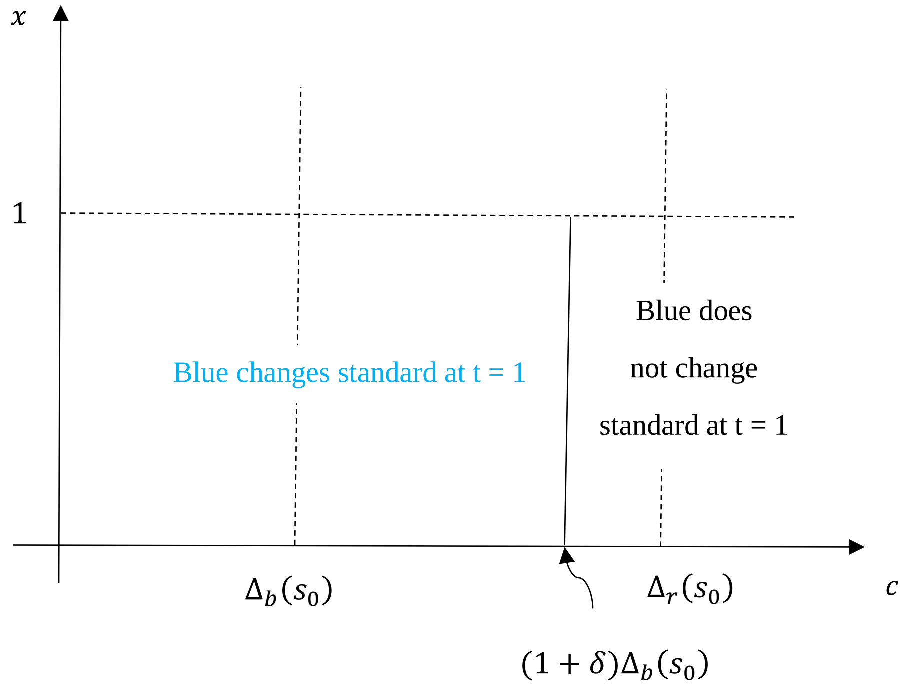


Figure T.1. Three thresholds when $s_0 \in \left[b, (1-\sqrt{\frac{\delta}{1+\delta}})r + \sqrt{\frac{\delta}{1+\delta}}b\right]$

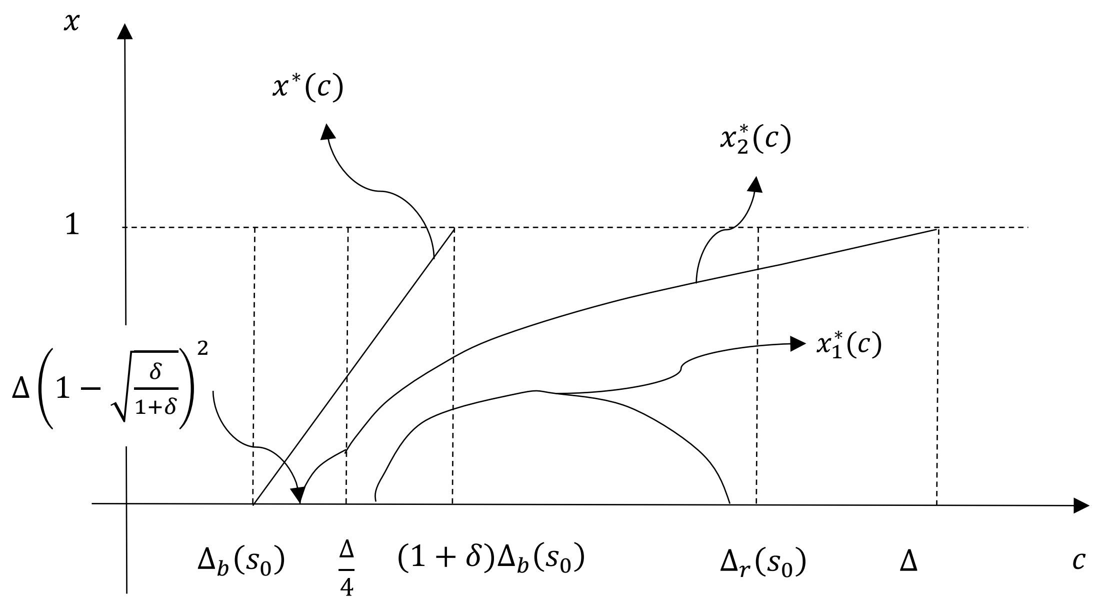


Figure T.2. Three thresholds when $s_0 \in \left[\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)r + \sqrt{\frac{\delta}{1+\delta}}b, \frac{1}{1+\sqrt{1+\delta}}r + \frac{\sqrt{1+\delta}}{1+\sqrt{1+\delta}}b\right]$

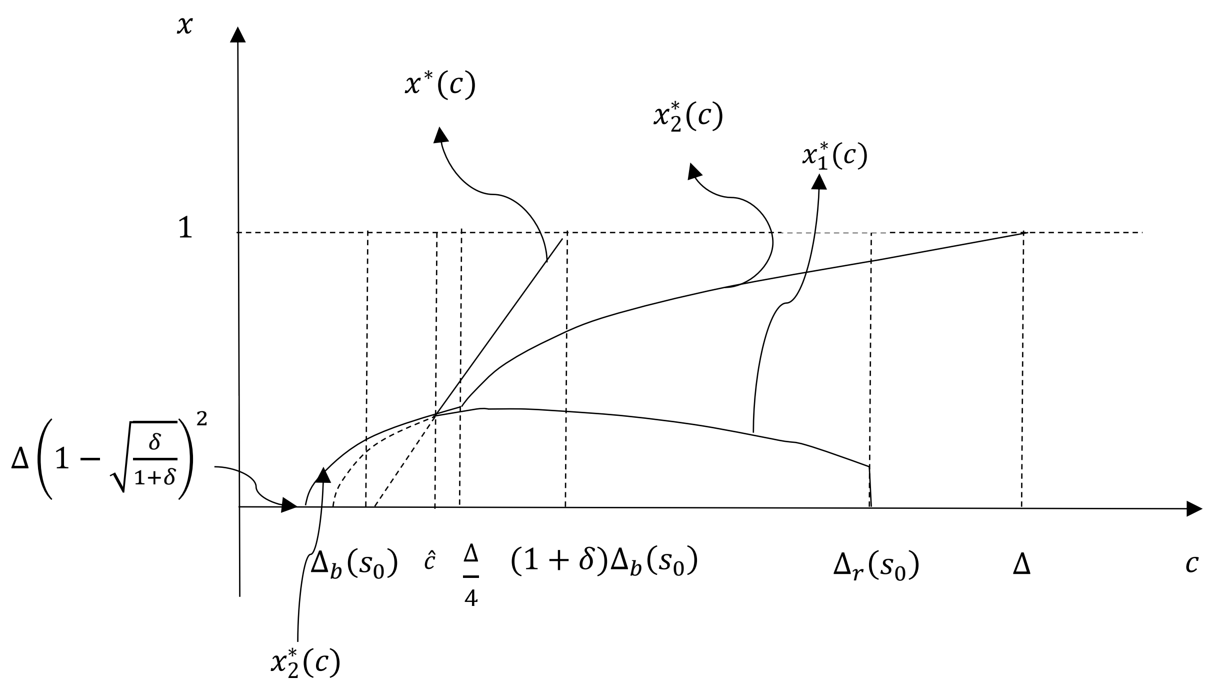


Figure 2.1.A. Majority decision at t = 1 when $s_0 \in \left[b, (1-\sqrt{\frac{\delta}{1+\delta}})r + \sqrt{\frac{\delta}{1+\delta}}b\right]$

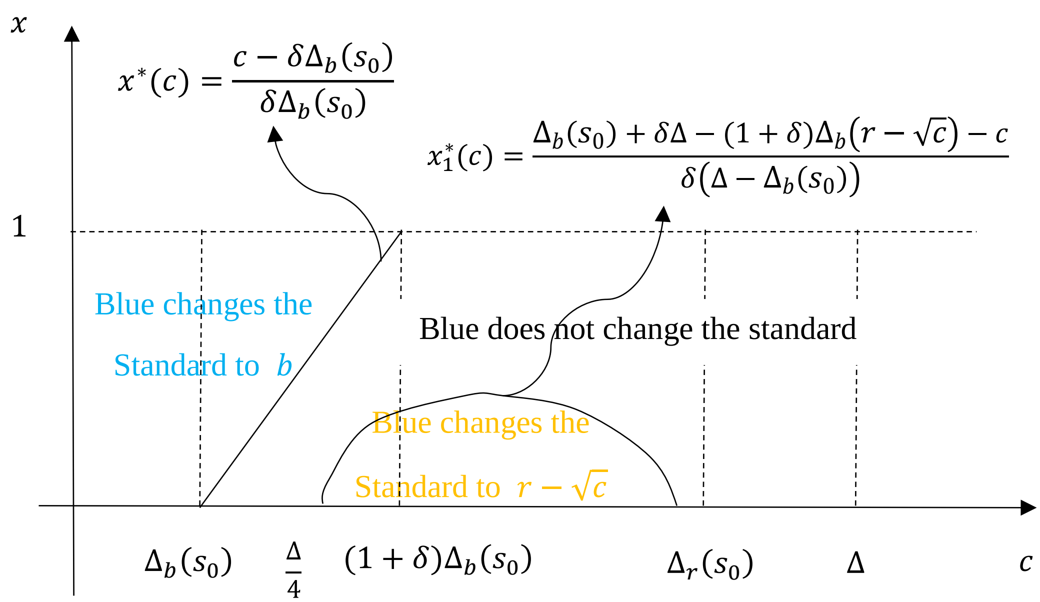


Figure 2.1.B. Majority decision when $s_0 \in \left[\left(1-\sqrt{\frac{\delta}{1+\delta}}\right)r + \sqrt{\frac{\delta}{1+\delta}}b, \frac{1}{1+\sqrt{1+\delta}}r + \frac{\sqrt{1+\delta}}{1+\sqrt{1+\delta}}b\right]$

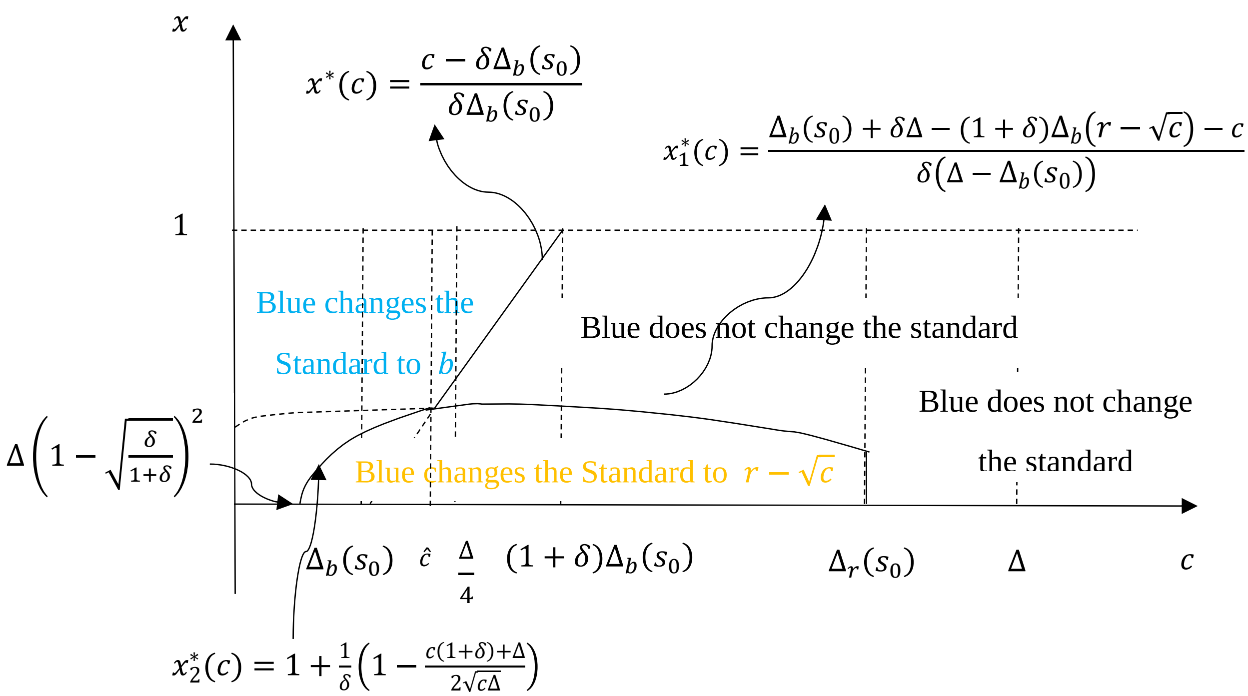
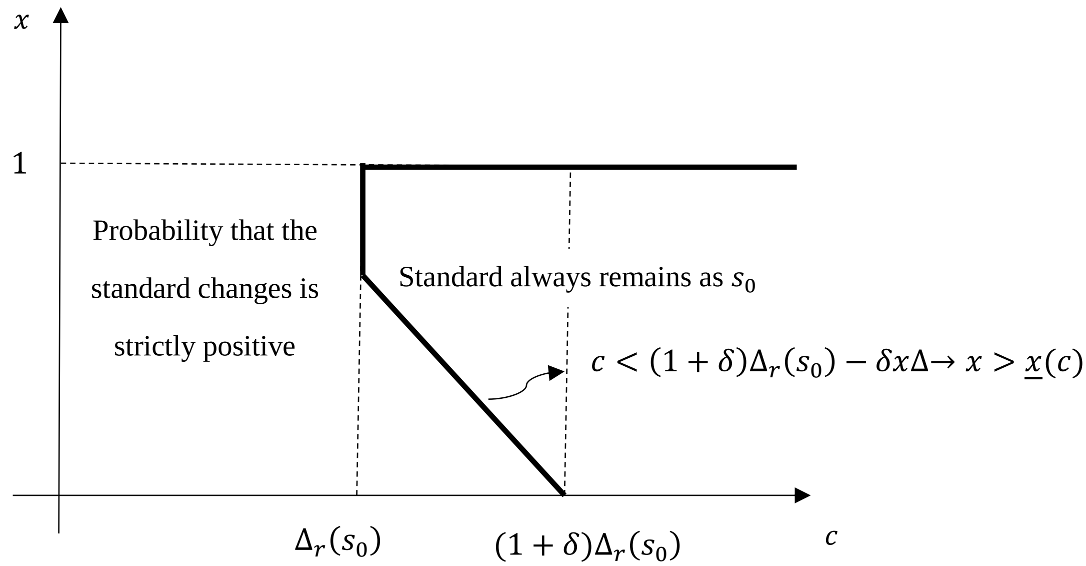


Figure 3 Areas of *Maximum Stability*

when $s_0 \in \left[b, \frac{r}{1+\sqrt{1+\delta(1+\delta)}} + \left(1 - \frac{1}{1+\sqrt{1+\delta(1+\delta)}}\right) b\right]$.



when $s_0 \in \left[\frac{r}{1+\sqrt{1+\delta(1+\delta)}} + \left(1 - \frac{1}{1+\sqrt{1+\delta(1+\delta)}}\right) b, \frac{r}{1+\sqrt{1+\delta}} + \left(1 - \frac{1}{1+\sqrt{1+\delta}}\right) b\right]$.