Aim and Scope of Electoral Districting

Andrei Gomberg

Romans Pancs

Tridib Sharma

ITAM

Disclaimers

- Preliminary work
- This is not a paper about "gerrymandering"!

Why districting?

- Periodically (every 10 years in the US) many countries redraw their electoral districts

- The process is frequently controversial

- May be used for electoral advantage either by parties, or by incumbents ("gerrymandering")

- It is easy to design an electoral system not requiring districts (e.g., proportional representation: Netherlands, Israel, etc.)

Why geographic districts?

(do not think local public goods, etc.)

- Residential choice is one of the most important (and costly to change) consumer decisions
- Not likely to be affected by electoral districting
- Localities ("precincts"), though heterogeneous in political preferences of their inhabitants, are mixed
- Tying up residential choice and districting allows the "districter" to allocate each voter to a representative

Questions

- Why districts?

- What legislatures are possible to obtain by districting?

- How to reveal objectives of the districter based on the observed districting outcomes *(in progress)*?

  (no this is still not about gerrymandering, unless…)
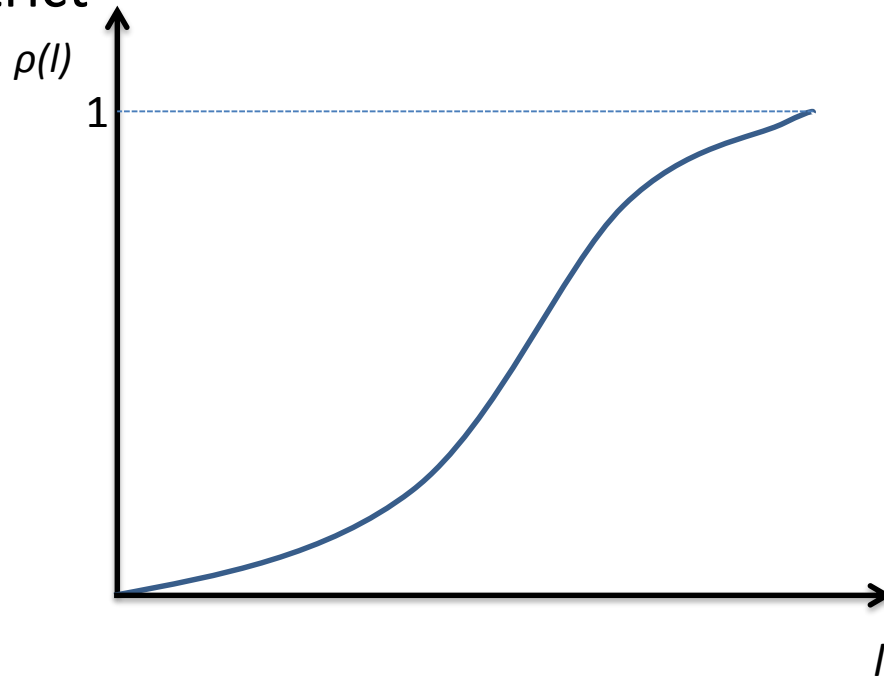
Literature

- **Social Choice:** Dodgson (1884), Black (1949), Chamberlin and Courant (1983) Monroe (1995), Chambers (2008, 2009), Puppe and Tasmadi (2010), Elkind et al. (2017, survey), etc., etc.

- **Partisan Districting** (gerrymandering), going back to the *reapportionment revolution* of the 1960s (*Baker v Carr 1962*): Vickrey (1961), Owen and Grofman (1988), Grofman et al. (2007), Sherstyuk (1998), Gilligan and Matsusaka (1999), Friedman and Holden (2008), Puppe and Tasnadi (2009), Chambers and Miller (2010), Chen and Rolden (2013), Tapp (2018), ….

- **Optimal and Strategic Districting:** Gilligan and Matsusaka (2006), Coate and Knight (2007), Gul and Pesendorfer (2010), Konishi and Pan (2018), Nagle (2019) ….

- **Legal Tests:** Grofman and King (2007), Wang (2015), McDonald and Best (2015), Best et al. (2018), Cervas and Grofman (2018), Grofman (2019), Stephanopoulos and McGee (2015, 2018), Chambers Miller and Sobel (2017)…. (even though *Rucho v Common Cause 2019*)

Model

- Binary ideology (*0* or *1* for each voter)
- Geographical continuum: voters uniformly distributed on *[0,1]*
- Affiliation function: *ρ(l)* – proportion of voters with ideology *1* at location *l* $\in$ *[0,1]*; *ρ(l)* – nondecreasing (WLOG: Hardy-Littlewood 1930)
- Districting: partition of *[0,1]* into *K* (odd) districts of equal size
- Each district *k* elects a *legislator* r_k with ideology equal to the district ideological mean (i.e., proportion of ideology 1 voters)
- Legislature: collection of legislators *r=(r_1, r_2,...,r_K)*
- Policy: ideology of the *median* legislator P= r_{med}

Precincts: affiliation function

- Proportion of population at location *l* with ideology 1: $\rho(l)$
- All voters at the same location have to be assigned to the same district

Geography

- We have geography (unlike most of the literature)
- Voters cannot be individually assigned to districts: they are movable only in precincts
- Our geography is unidimensional and we do not care about "contiguity", "compactness", etc. (unlike most papers that actually talk about geography)
- Gain: tractability
- Loss: does not seem to matter much (Sherstyuk 1998)

Geography

- We have geography (unlike most of the literature)
- Voters cannot be individually assigned to districts: they are movable only in precincts
- Our geography is unidimensional and we do not care about "contiguity", "compactness", etc. (unlike most papers that actually talk about geography)
- Gain: tractability
- Loss: does not seem to matter much (Sherstyuk 1998)
- **Anyway: this is not a paper on gerrymandering!**

First results

The set of policies implementable through drawing K equal-sized districts is

- a strict subset of *(0,1)*

- contains the mean $R= \int \rho(l) \; dl$

- increasing in K

- increasing in the mean-preserving spread of $\rho(l)$

Benchmark: no geography

- Voters can be directly assigned to any district
- Equivalent to the furtherst spread of ρ
  - $\rho(l)=0$, $l \leq 1-R$
 - $\rho(l)=1$, $l >1-R$

Proportional representation

- Monroe (1995): if voters only care about how far their policy is from that of their district representative, maximizing aggregate utility requires sorting by ideology

-
  assuming away integer problems

  - (1-R)K ideology 0 representatives
  - *RK ideology 1 representatives*
  - *policy equal to the ideal policy of the majority*

- Caplin and Nalebuff (1997): may be implemented by free mobility: let people choose their representatives

What have we learnt

- Proportional representation does great on having voters represented by legislators of their choice

- Extreme on policy: minority may fare badly

- Assigning voters to districts by precinct may moderate policy: may be good if the districter is benevolent

- Downside: creates scope for districting malfeasance

Back to geography

- Consider arbitrary $\rho(l)$
- Which legislatures are feasible given $\rho(l)$?
- Benchmark: perfectly sorted districts
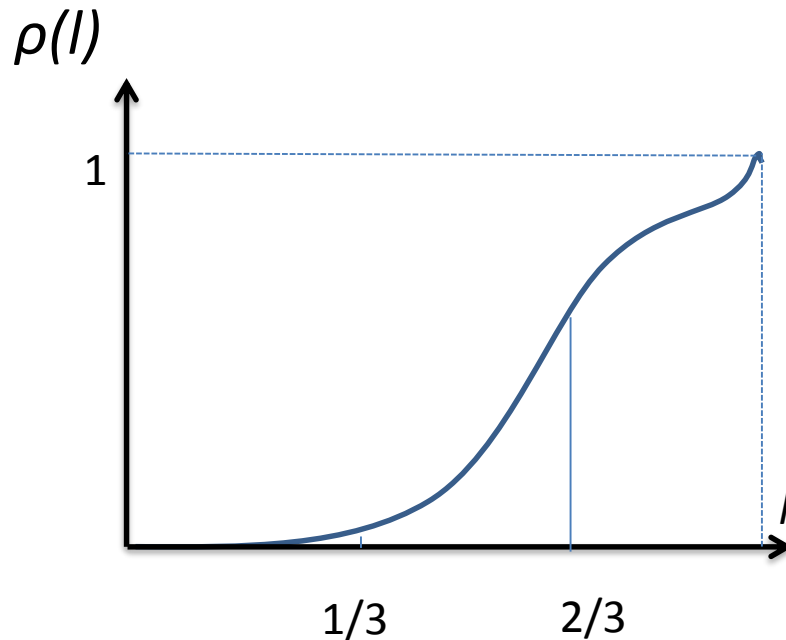
Perfectly sorted district map

- Let μ_i be the mean of the ith district on this map

- This map with the legislature given by $r_i = \mu_i$ maximizes total social welfare if voters only care about their own representative

- This map is the "most informative" (Athey and Levine 2018) feasible map: under it, if I know in which district the voter is assigned I can learn the most about his ideology
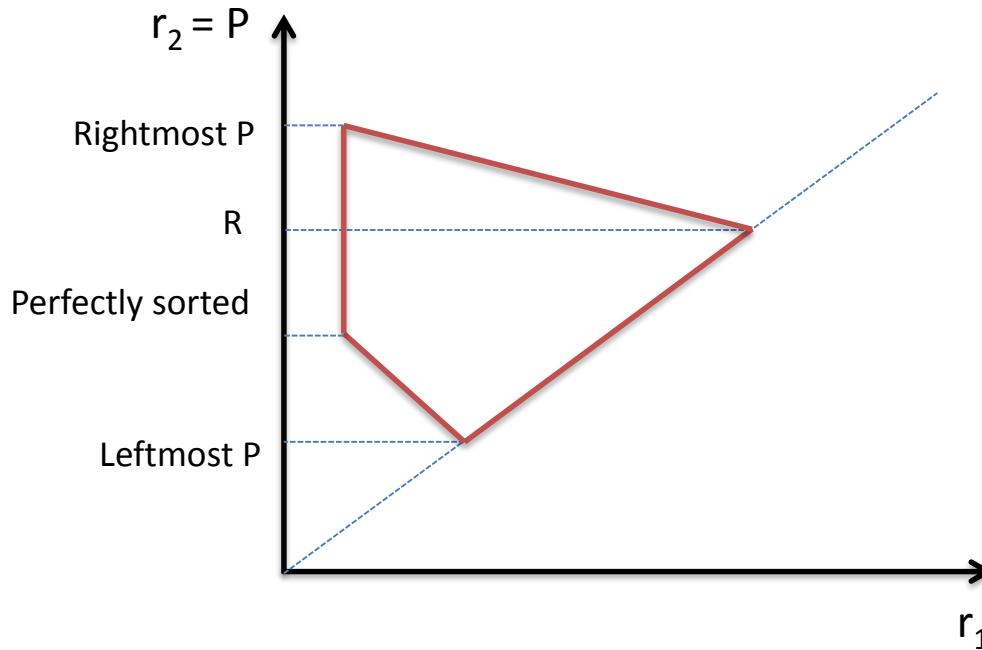
Feasible legislatures

- Theorem: the following are equivalent:
  - A legislature is feasible
  - A legislature is induced by a less informative map than the perfectly sorted district map
  - A legislature belongs to a (K-1)-dimensional polytope with vertices defined by a simple formula based on μ_i
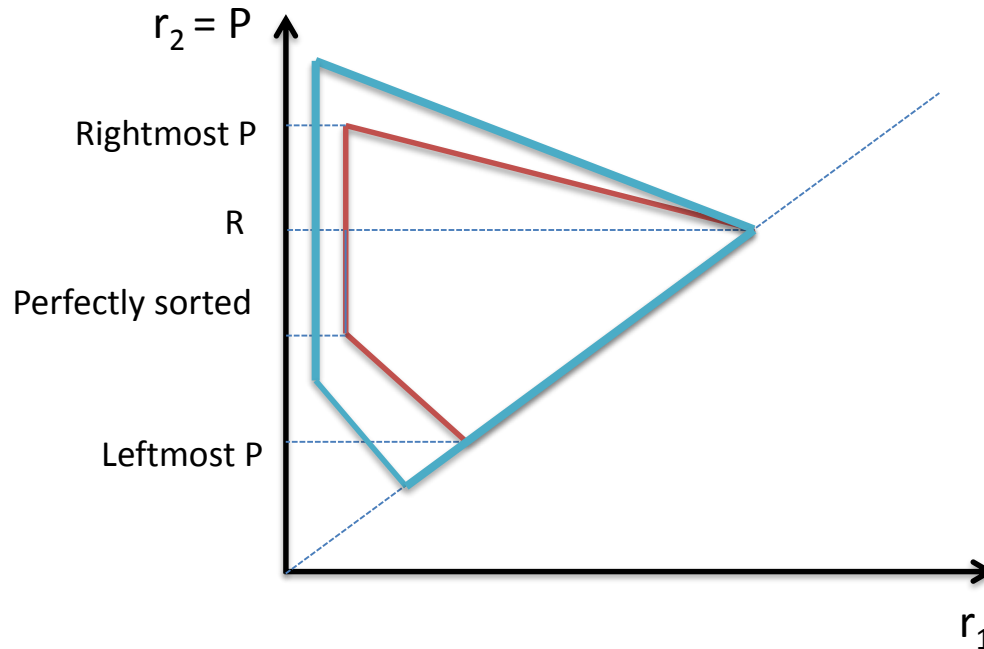
Feasible legislatures (K=3)

- Normalization: $r_1 \leq r_2 \leq \dots \leq r_K$

Increasing precinct heterogeneity

- A mean-preserving spread of $\rho(l)$

Optimal districting: social welfare approach

- Suppose people care about both the identity of their representative *and* about the policy implemented

$$U(a, r_i, P) = -\gamma(a - P)^2 - (1 - \gamma)(a - r)^2$$

- Utilitarian social welfare maximizing districter assigns each precinct *l* to a district with representative *r(l)* to maximize

$$-\gamma \int [(1-P)^2 \rho(l) + P^2(1-\rho(l))]dl$$

$$-(1-\gamma) \int [(1-r(l))^2 \rho(l) + (r(l))^2(1-\rho(l))]dl$$

Social welfare depends only on legislature

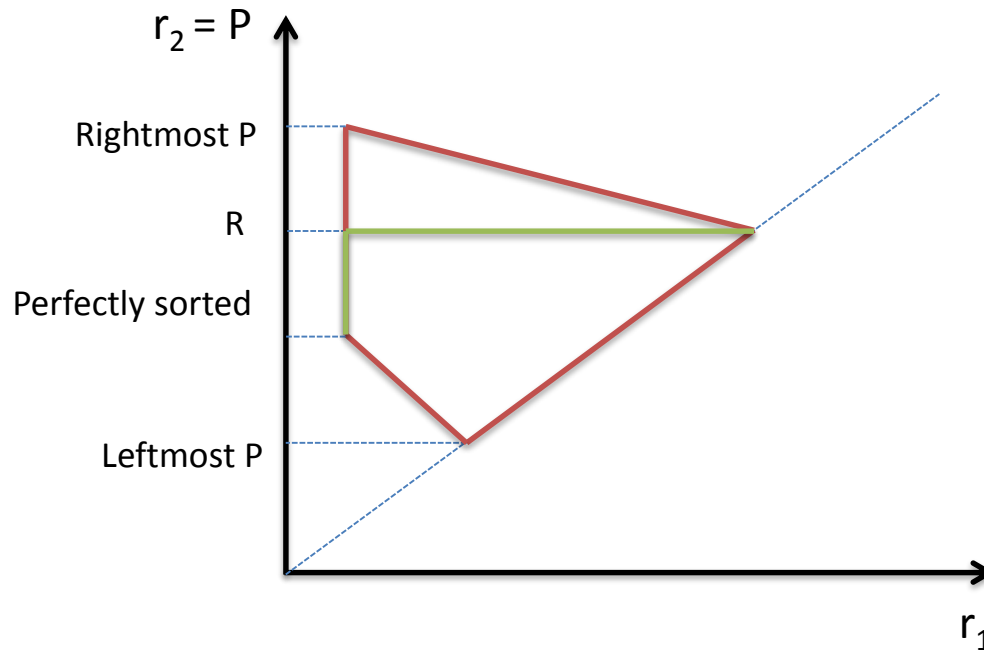- Proposition: maximum welfare is achieved in the feasible legislature that maximizes

$$\gamma(2Rr_{med} - r^2{}_{med}) + \frac{1 - \gamma}{K}\sum_k(r_k)^2$$

Importance: set of feasible maps is hard to describe; this proposition means we can work on the polytope of feasible legislatures alone

Possibly optimal legislatures (K=3)

- Depending on γ

Tools of the trade: *Pack* and *Split*

- To do the best on policy *(γ=1)*: split everyone
- To do the best on representation *(γ=0)*: pack everyone
- To get extreme policies: split the side you try to move the policy towards, pack the other side
- More things could be done with more districts: but they all *pack* and *split*

Introducing turnout

- What if turnout is (predictably) variable, but the target population of a district still depends on the census population, not on the turnout?
- $\tau(l)$ – turnout share at location l
- generally not correlated with $\rho(l)$
- We loose the polytope structure of the feasible legislature set
- "Pack and split" based on vote share are no longer the only useful tools

Drawing districts by the score

(just a sketch)

- How to move the policy to the right?

- For any target median policy P define the score:

$$S(l;P) = (\rho(l) - P)\tau(l)$$

- Build the districts based on the score

- This would mix high turnout right-wing precincts with low turnout left-wing precincts

What have we got

- Pure proportional representation does great on giving each voter the representative he likes, but hurts many on policy
- "At large" elections would do good on policy: but do not give voters good representation
- Geographic districts allow the system designer to achieve a compromise between the two objectives
- How efficient a tool it is depends on affiliation between geography and policy $\rho(l)$
- *Tradeoff between welfare maximization and malfeasance*

Where from now: work in progress

- Developing measures of "improper" redistricting incorporating geographic information
  - "partisan gerrymandering"
  - "pro-incumbency gerrymandering"
- Comparing with empirical maps *(we are collecting the data)*